1.3 Exponential Functions

Warm-Up/Review

- Show algebraically that $f(x) = 4x^3 - 3x$ is an odd function.

A function is odd if $f(-x) = -f(x)$

$$\begin{aligned} f(-x) &= 4(-x)^3 - 3(-x) \\ &= 4(-1)^3(x)^3 - 3(-1)(x) \\ &= 4(-1)(x)^3 - 3(-1)(x) \\ &= (-1)(4x^3 - 3x) \\ &= -f(x) \end{aligned}$$

- Show algebraically that $f(x) = \frac{x}{2x^3+x}$ is an even function.

A function is even if $f(-x) = f(x)$

$$\begin{aligned} f(-x) &= \frac{-x}{2(-x)^3+(-x)} \\ &= \frac{-1(x)}{-1(2x^3+x)} \\ &= \frac{x}{2x^3+x} = f(x) \end{aligned}$$

Investigation

- Graph the function $y=a^x$ for a=2,3,5, in a [-5,5] by [-2,5] viewing window.
- For what values of x is it true that $2^x<3^x<5^x$? $x > 0$
- For what values of x is it true that $2^x>3^x>5^x$? $x < 0$
- For what values of x is it true that $2^x=3^x=5^x$? $x = 0$

Investigation

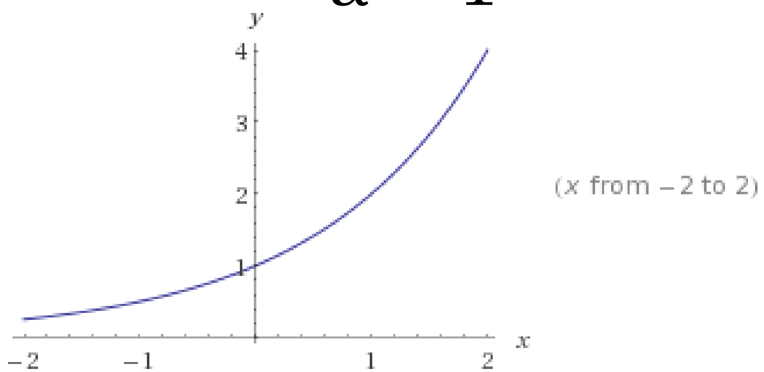
- Graph the function $y=(1/a)^x = a^{-x}$ for $a=2,3,5$, in a $[-5,5]$ by $[-2,5]$ viewing window.
- For what values of x is it true that $2^x<3^x<5^x$? $x < 0$
- For what values of x is it true that $2^x>3^x>5^x$? $x > 0$
- For what values of x is it true that $2^x=3^x=5^x$? $x = 0$

Exponential Functions

- Exponential functions have the form
- $f(x) = a^x$, $a>0$, and $a\neq1$
- where a is the base
- Domain is $(-\infty, \infty)$
- Range is $(0, \infty)$

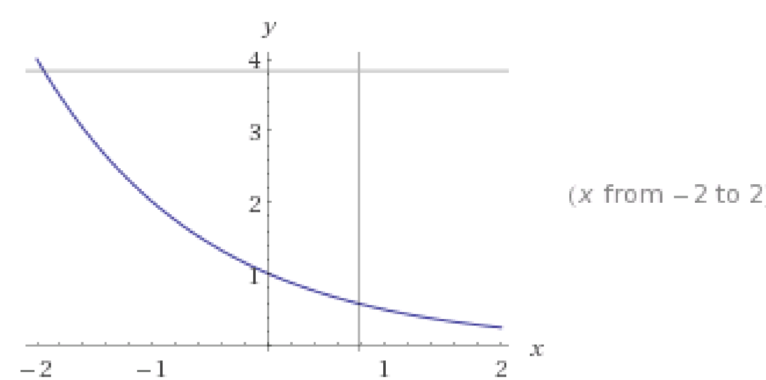
Plots:

$a > 1$



$0 < a < 1$

Plots:



Exponential Functions – rules of exponents:

- If a>0 and b>0, the following are true for all real numbers x and y:
- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^x b^x = (ab)^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Exponential Functions – Applications

- Exponential growth – population growth, interest & investments, bacterial growth
- $y=ka^x$, $k>0$ and $a>1$
- Exponential decay – half-life of radioactive elements
- $y=ka^x$, $k>0$ and $0<a<1$

Example: Population Growth

- The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

$P(t) = 6250(1.0275)^t$
Pop in 1915 = P(25) = 12,315
Pop in 1940 = P(50) = 24,265

- a) Estimate the population in 1915 and 1940
- b)Approximately when did the population reach 50,000?

Graph y1=P(t) and y2=50,000 and find the intersection.
t=76.75 years

Example: Half-Life

- Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially. When will there be only 1 gram of the substance remaining?
- $A(t)=5(1/2)^{t/20}$ models the mass in grams after t days.
- Solve graphically

Example: Compound Interest

- $y = A(1 + \frac{I}{n})^{nt}$
- Where n is the number of times interest is compounded per year
- I is the interest rate (decimal)
- A is the starting amount

The number e

- The functions $y=e^x$ and $y=e^{-x}$ are often used as models of exponential growth or decay.
- *Interest compounded continuously*
- $\mathbf{y=Pe^{rt}}$
- P=initial investment
- e=2.71828…
- r=interest rate (in decimal form)
- t=time in years

Assignment

- P. 24 # 1-7, 10, 11, 13, 16, 18, 23-33 odd
- Due Monday
- Quiz tomorrow – lines, slope, functions & graphs, domain, range, symmetry, piecewise functions, combining functions!