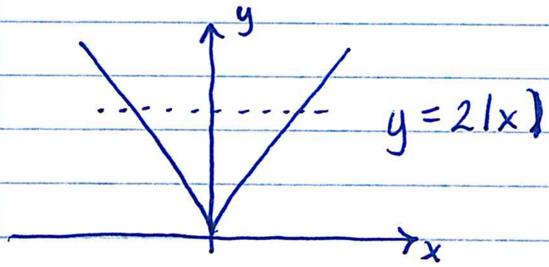
KEY

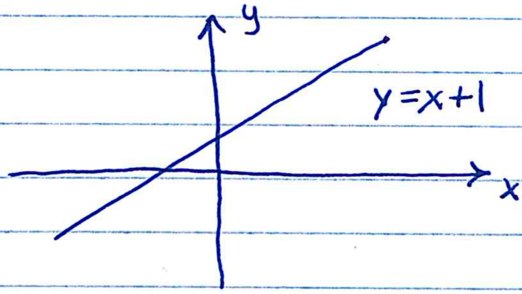
1-5A: p 39 # 1-12, 14, 19, 21, 23, 24

One-to-one function means $f(a) \neq f(b)$ whenever $a \neq b$.
In other words - the graph of a one to one function $y = f(x)$ can intersect any horizontal line at most once (the horizontal line test).

① 

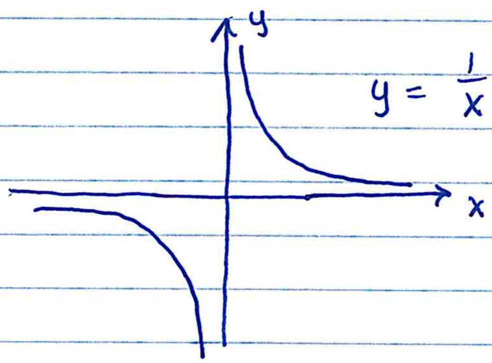
Not one-to-one

fails the horizontal line test (2 different values of x have the same y value.)

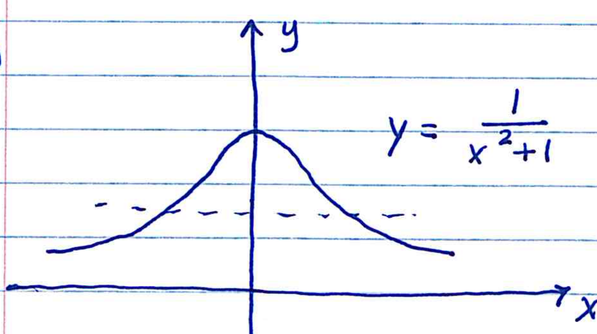
② 

Yes one-to-one

each value of x has a unique value of y.

③ 

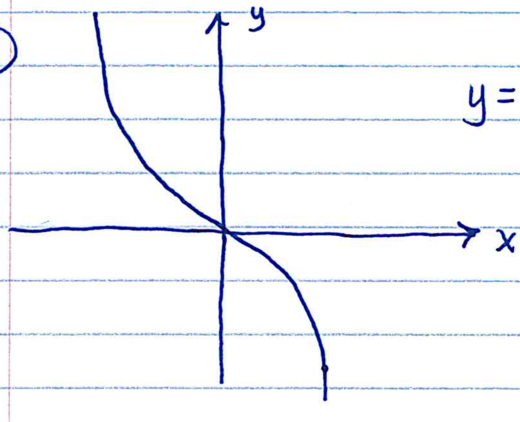
Yes one-to-one

④ 

No not one-to-one

horizontal line intersects the graph more than once.

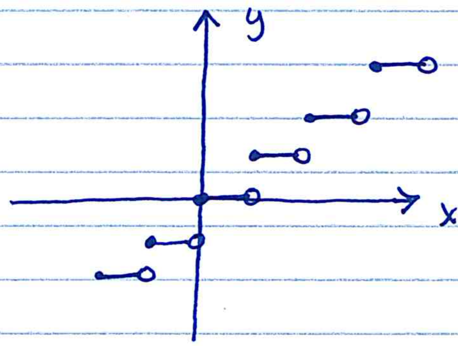
(5)



$y = -3x^3$ Yes, one-to-one

You can verify this on your calculator by zooming in around $x = 0$

(6)

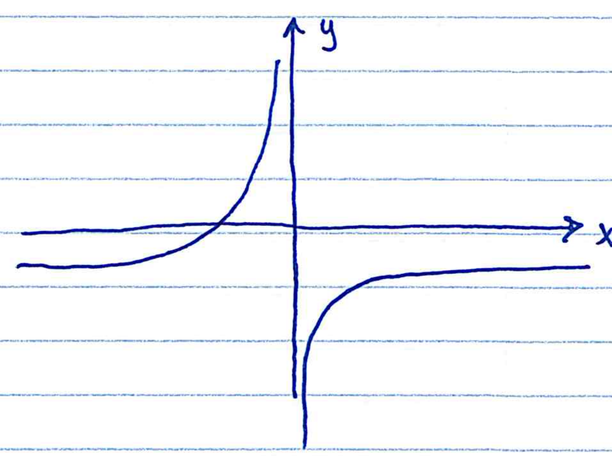


$y = \text{int } x$ No not one-to-one

(7). A function that is one-to-one on a domain has an inverse function

$$y = \frac{3}{x-2} - 1$$

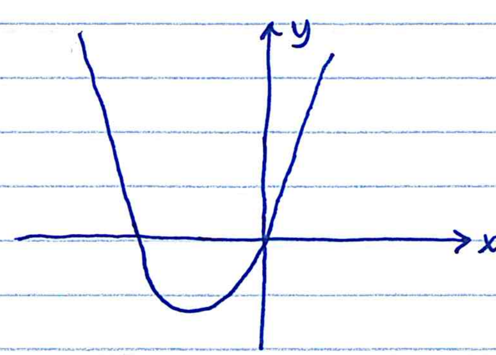
graph



the function is one-to-one so the function has an inverse.

Yes

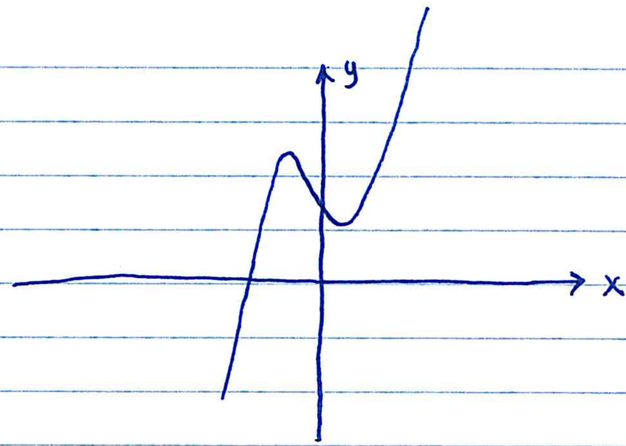
(8) $y = x^2 + 5x$



No inverse.
The function is not one-to-one

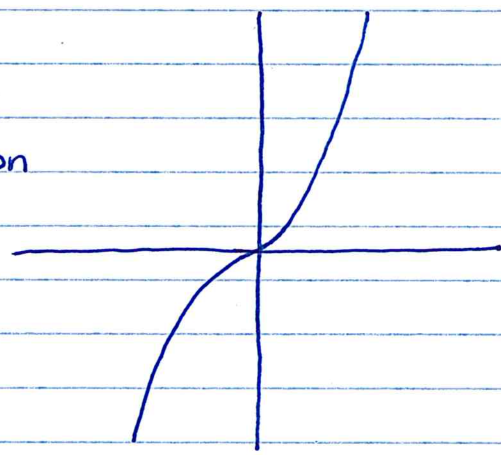
(9) $y = x^3 - 4x + 6$

No inverse
The function is
not one to one.



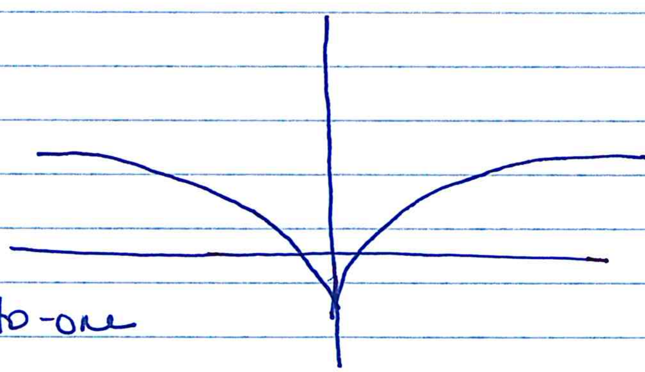
(10) $y = x^3 + x$

Yes the function
is one-to-one
so it has
an inverse.



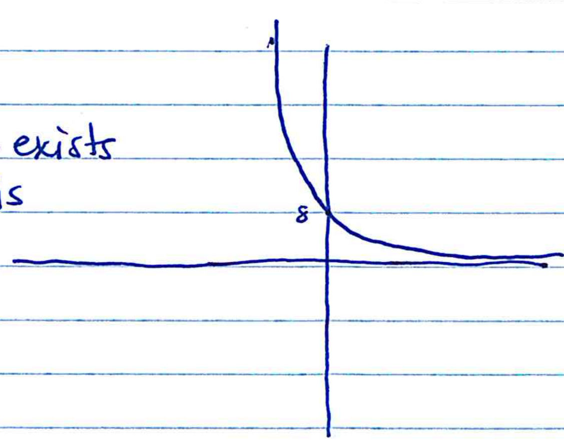
(11) $y = \ln x^2$

No inverse
the function
is not one-to-one



(12) $y = 2^{3-x}$

Yes an inverse exists
the function is
one-to-one.



(14) $f(x) = 5 - 4x$

$y = 5 - 4x$

① solve for x

$y = 5 - 4x$

$+4x \qquad +4x$

$4x + y = 5$

$4x = -y + 5$

$x = -\frac{1}{4}y + \frac{5}{4}$

② swap x and y

$y = -\frac{1}{4}x + \frac{5}{4}$

$f^{-1}(x) = -\frac{1}{4}x + \frac{5}{4}$

check $f \circ f^{-1}(x) = f\left(-\frac{1}{4}x + \frac{5}{4}\right)$

$= 5 - 4\left(-\frac{1}{4}x + \frac{5}{4}\right)$

$= 5 + x - 5$

$= x$

$f^{-1} \circ f(x) = f^{-1}(5 - 4x)$

$= -\frac{1}{4}(5 - 4x) + \frac{5}{4}$

$= -\frac{5}{4} + x + \frac{5}{4}$

$= x$

$f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$

(19) $f(x) = -(x-2)^2 \quad \underbrace{x \leq 2}$

$y = -(x-2)^2$

① solve for x

$-y = (x-2)^2$

$-\sqrt{-y} = x - 2$

we need the $-\sqrt{-y}$ b/c $x - 2 \leq 0$ for our domain.

$x - 2 = -\sqrt{-y}$

$x = 2 - \sqrt{-y}$

② swap x and y

$y = 2 - \sqrt{-x}$

$f^{-1}(x) = 2 - \sqrt{-x}$ for $\underline{x \leq 0}$

Domain of $f^{-1}(x)$ $x \leq 0$

check:

$f \circ f^{-1}(x) = f(2 - \sqrt{-x})$

$= -(\cancel{2} - \sqrt{-x} - \cancel{2})^2$

$x \leq 0$ $\quad = -(-\sqrt{-x})^2$

$= -(-x)$

$= x$

$f^{-1}(f(x)) = f^{-1}\left(-(x-2)^2\right)$

$= 2 - \sqrt{+(+(x-2)^2)}$

$= 2 - \sqrt{(x-2)^2}$

$= 2 - |x-2|$

for ~~when~~ $x \leq 2 \quad |x-2| \leq 0$

so we need $-(x-2)$

$= 2 - (-(x-2))$

$= 2 + x - 2$

$= x$

(21) $f(x) = \frac{1}{x^2} \quad x > 0$

$y = \frac{1}{x^2}$

① solve for x

$x^2 = \frac{1}{y}$

$x = \pm\sqrt{\frac{1}{y}}$ since $x > 0$ we want $+\sqrt{\frac{1}{y}}$

$x = \sqrt{\frac{1}{y}}$

② swap x and y

$y = \frac{1}{\sqrt{x}} \quad x > 0$

$f^{-1}(x) = \frac{1}{\sqrt{x}}$

$f \circ f^{-1}(x) = f\left(\frac{1}{\sqrt{x}}\right) \quad x > 0$

$= \frac{1}{\left(\frac{1}{\sqrt{x}}\right)^2}$

$= \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

$f^{-1} \circ f(x) = f^{-1}\left(\frac{1}{x^2}\right) \quad x > 0$

$= \frac{1}{\sqrt{\frac{1}{x^2}}}$

$= 1 \cdot \sqrt{\frac{x^2}{1}} = \sqrt{x^2} = |x|$

$= x$

$x > 0$

(23) $f(x) = \frac{2x+1}{x+3}$

① solve for x

$y = \frac{2x+1}{x+3}$

$y(x+3) = 2x+1$

$xy + 3y = 2x + 1$

$xy - 2x = -3y + 1$

$x(y-2) = -3y + 1$

$x = \frac{-3y+1}{y-2}$

② swap x and y

$y = \frac{-3x+1}{x-2}$

$f^{-1}(x) = \frac{-3x+1}{x-2}$

$f \circ f^{-1}(x) = f\left(\frac{-3x+1}{x-2}\right)$

$= \frac{2\left(\frac{-3x+1}{x-2}\right) + 1}{\frac{-3x+1}{x-2} + 3}$

$= \frac{\frac{-6x+2+x-2}{x-2}}{\frac{-3x+1+3(x-2)}{x-2}}$

$= \frac{\cancel{-5}x}{\cancel{x-2}} \cdot \frac{\cancel{x-2}}{\cancel{-5}}$

$= x$

see next page

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+1}{x+3}\right)$$

$$= \frac{-3\left(\frac{2x+1}{x+3}\right)+1}{\frac{2x+1}{x+3}-2}$$

$$= \frac{\frac{-6x-3+x+3}{x+3}}{\frac{2x+1-2x-6}{x+3}}$$

$$= \frac{-5x}{x+3}\cdot\frac{x+3}{-5}$$

$$\underline{\underline{= x}}$$

(24) $f(x) = \frac{x+3}{x-2}$

$y = \frac{x+3}{x-2}$

① solve for x

$y(x-2) = x+3$

$xy - 2y = x+3$

$xy - x = 2y+3$

$x(y-1) = 2y+3$

$x = \frac{2y+3}{y-1}$

② swap x and y

$y = \frac{2x+3}{x-1}$

$f^{-1}(x) = \frac{2x+3}{x-1}$

$$f\circ f^{-1}(x) = f\left(\frac{2x+3}{x-1}\right)$$

$$= \frac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}$$

$$= \frac{\frac{2x+3+3x-3}{x-1}}{\frac{2x+3-2x+2}{x-1}}$$

$$= \frac{5x}{x-1}\cdot\frac{x-1}{5} = \underline{\underline{x}}$$

$$f^{-1}\circ f(x) = f^{-1}\left(\frac{x+3}{x-2}\right)$$

$$= \frac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1} = \frac{\frac{2x+6+3(x-2)}{x-2}}{\frac{x+3-x+2}{x-2}}$$

$$= \frac{5x}{x-2}\cdot\frac{x-2}{5}$$

$$\underline{\underline{= x}}$$