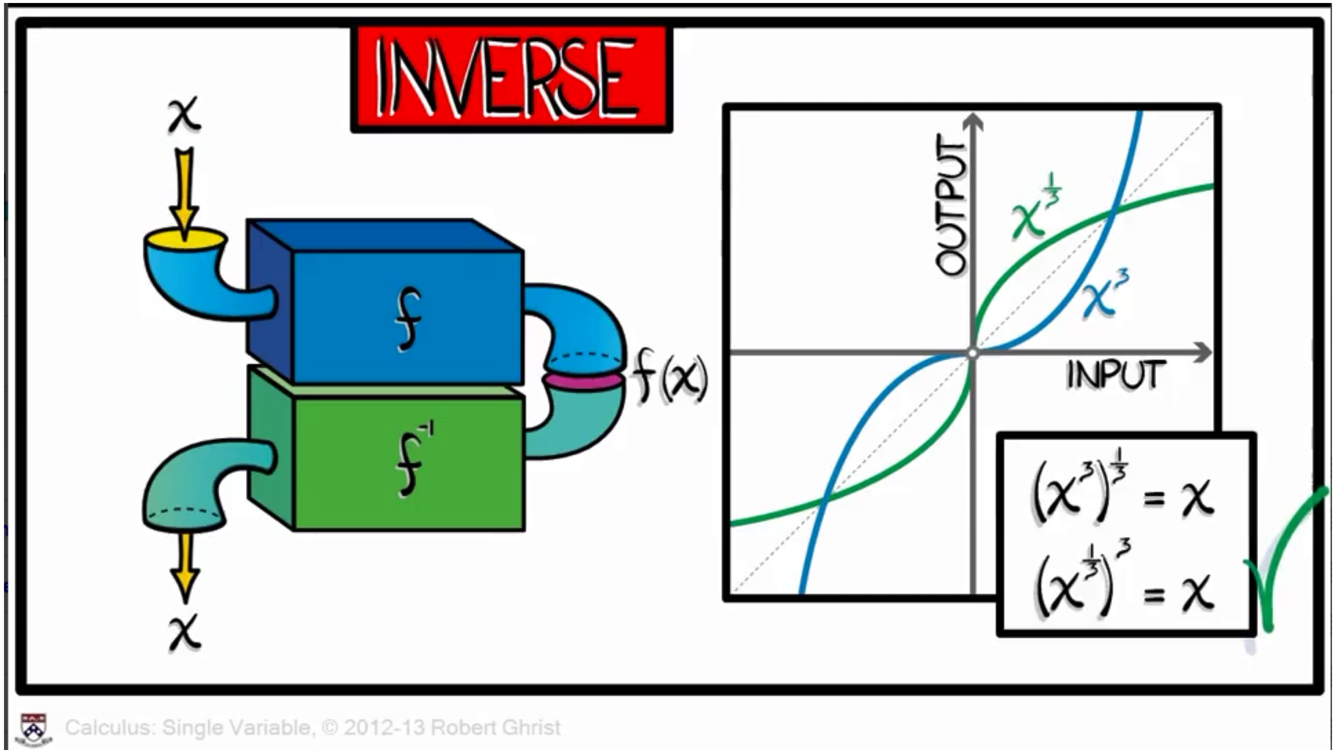
1.5A Functions and Inverses

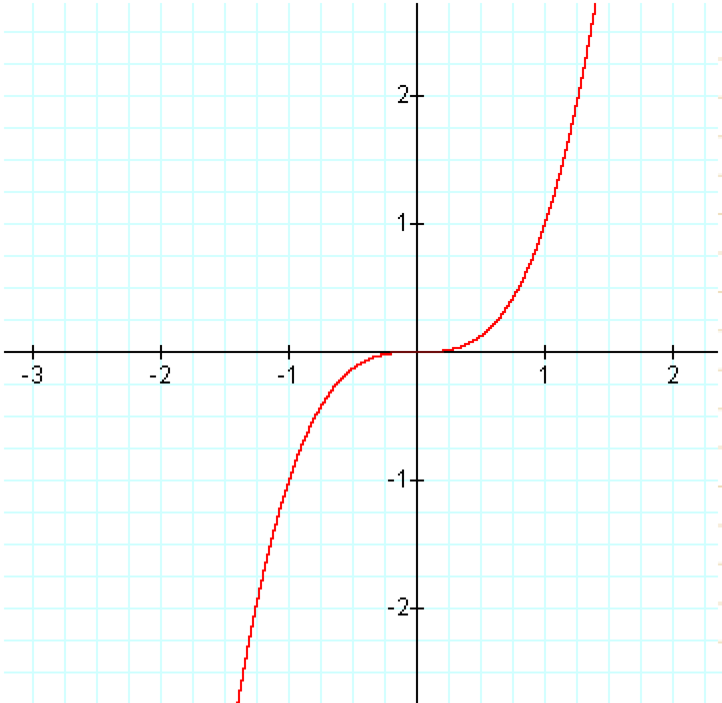
INVERSE FUNCTION

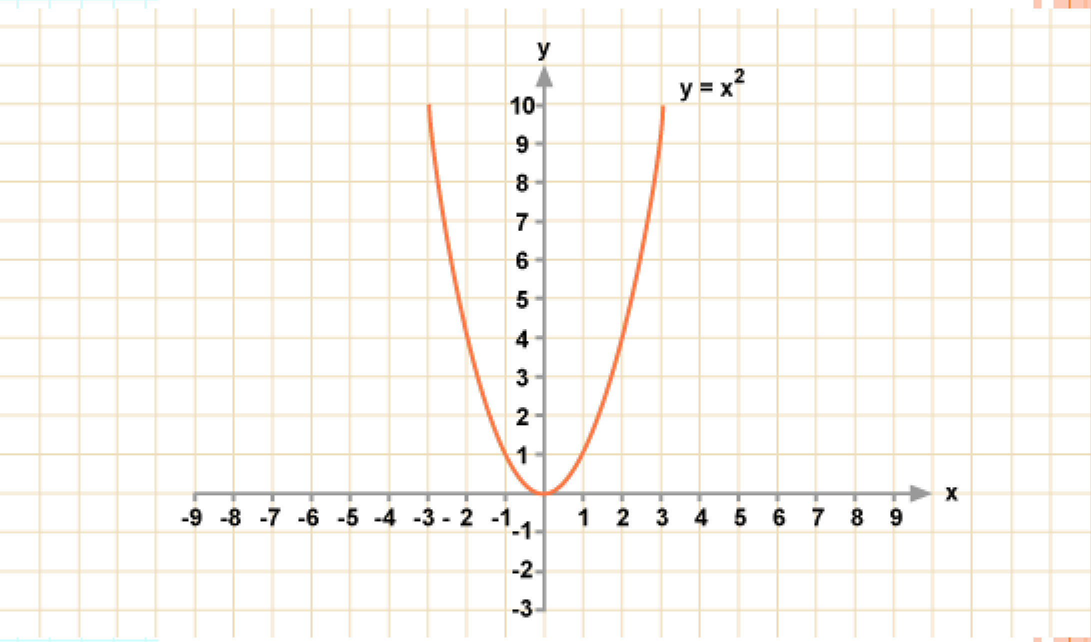
- A **function**, f, takes an input (x), does something to it, and produces an output, f(x).
- An **inverse function** is a function that will undo anything that the original function does.



Inverse function - Notation

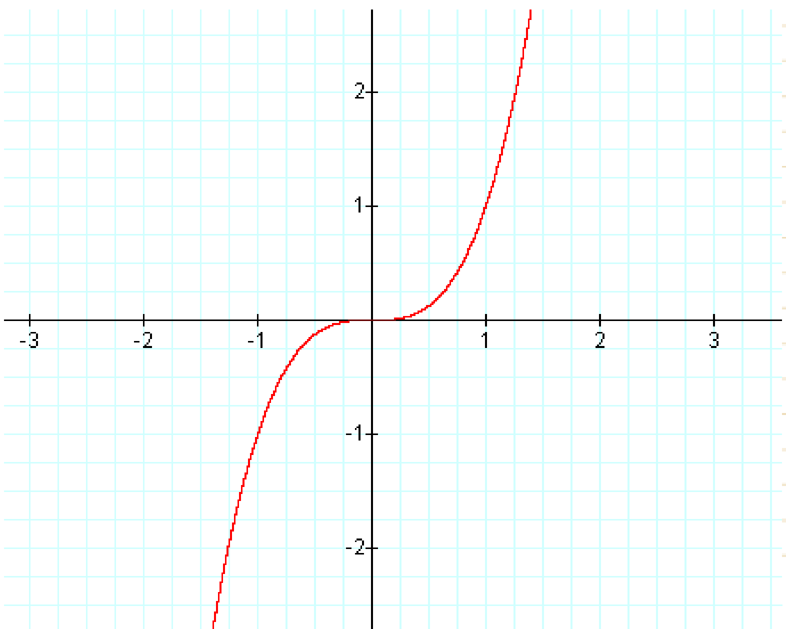
- $f(x)$ denotes the function
- $f^{-1}(x)$ denotes the inverse of f(x)
- Not all functions have inverses!

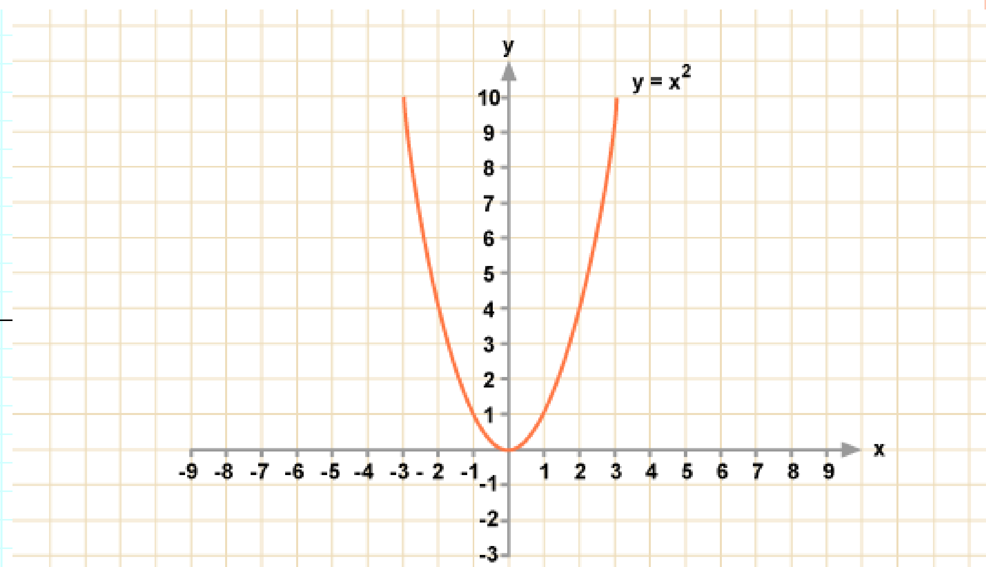




ONE-TO-ONE FUNCTION

- A function $f(x)$ is **one-to-one** on a domain D if f(a) ≠ f(b) whenever a ≠ b.
- Each input has a unique output
- We can use the *horizontal line test*





One-to-One Function

- Use your calculator to determine which functions are one-to-one:
- f(x) = |x| No
- g(x) = $\sqrt{x}$ Yes
- h(x) = 3x + 2 Yes
- t(x) = $x^3 - 4x$ No

Finding Inverses

- The result of composing a function and its inverse in either order is the identity function
- $f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$

Finding Inverses

- Make sure the function is one-to-one.
- If it is not one-to-one, you can restrict the domain to make it one-to-one.
- Solve the equation $y = f(x)$ for x in terms of y
- Switch x and y. Your result will be $y = f^{-1}(x)$
- Verify that $f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$

Finding Inverses – Example 1

- f(x) = -2x + 4
- *The function is a line and is one-to-one*
- *Solve the equation* $y = f(x)$ *for* x *in terms of* y
- $y = -2x + 4$
- $2x = 4 - y$
- $x = 2 - \left(\frac{1}{2}\right) y$
- *Switch x and y. Your result will be* $y = f^{-1}(x)$
- $y = 2 - \left(\frac{1}{2}\right) x$
- $f^{-1}(x) = 2 - \left(\frac{1}{2}\right) x$
- Verify $f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$

Finding Inverses – Example 2

- $f(x) = x^2$ for x≤0
- *The function x^2 is not one-to-one, but when the domain is restricted to x≤0, it is one-to-one*
- *Solve the equation $y = f(x)$ for x in terms of y*
- $y = x^2$
- $x = \pm\sqrt{y}$ since our domain is x ≤0, we need to use -$\sqrt{y}$
- $x = -\sqrt{y}$
- *Switch x and y. Your result will be $y = f^{-1}(x)$*
- $y = -\sqrt{x}$
- $f^{-1}(x) = -\sqrt{x}$ the domain for f^{-1} is x≥0
- Continue to verification next page

Finding Inverses – Example 2

- $Verify\ \ f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$
- $f \circ f^{-1}(x) = x \quad\quad for\ x \geq 0\ (domain\ of\ f^{-1})$
- $f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

- $Verify\ \ f^{-1} \circ f(x) = x\ \ for\ x \leq 0\ (domain\ of\ f)$
- $f^{-1}(x^2) = \left(-\sqrt{x^2}\right) = -|x| = -(-x) = x$

  recall $|x| = -x$ *for* $x < 0$

Assignment

- P. 39 #1-12, 14, 19, 21, 23, 24