Continuity

Warm-Up

- Find $\lim_{x \to -1} \frac{3x^2 - 2x + 1}{x^3 + 4}$
- Let $f(x) = \begin{cases} x^2 - 4x + 5, & x < 2 \\ 4 - x, & x \geq 2 \end{cases}$

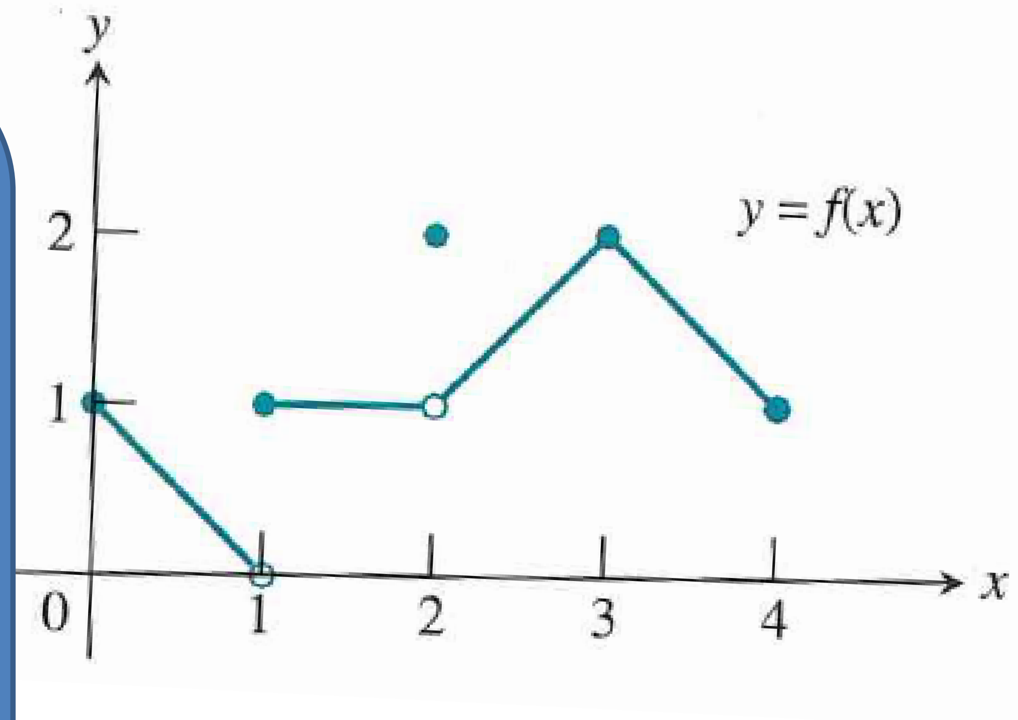
Find $\lim_{x \to 2^-} f(x)$

$\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x)$

$f(2)$

Where is f(x) continuous?



Can you see any general “rules” for continuity?

Requirements for Continuity

- $f(c)$ is defined
- $\lim_{x \to c} f(x)$ exists
- $f(c) = \lim_{x \to c} f(x)$

- For an interior point, the two sided limit must exist.
- For an endpoint, continuity requires only the one-sided limit.

Requirements for Continuity

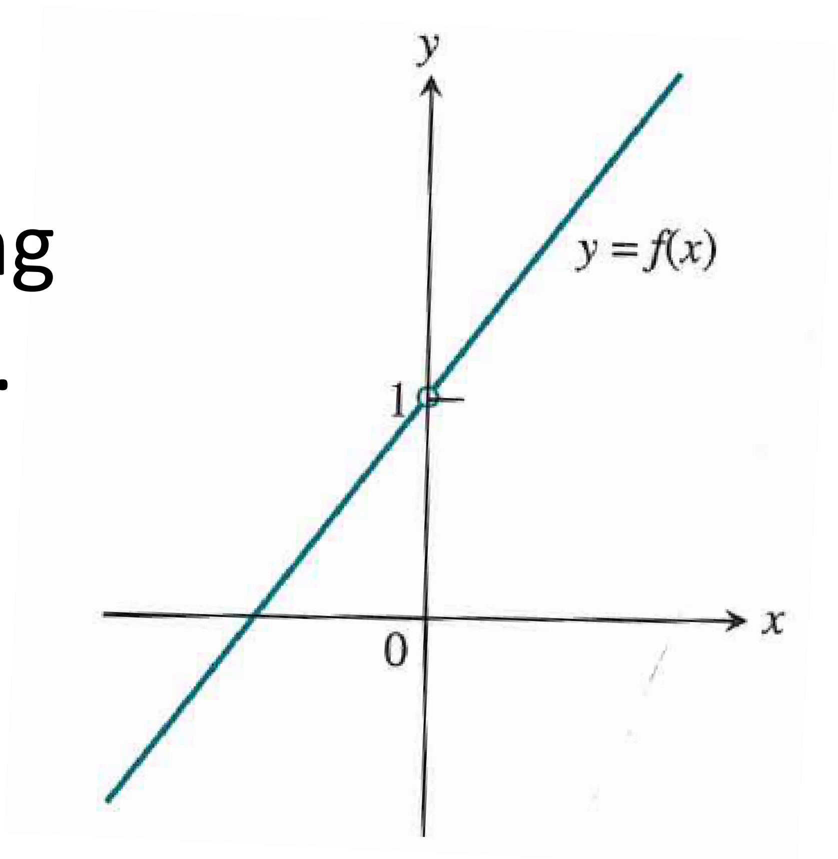
- If a function is not continuous at a point c, we say that *f* is **discontinuous** at c, and c is a **point of discontinuity** of f.
- There are 4 types of discontinuities…

Types of Discontinuity

- Removable discontinuity (hole)
- Jump discontinuity
- Infinite discontinuity (vertical asymptote)
- Oscillating discontinuity

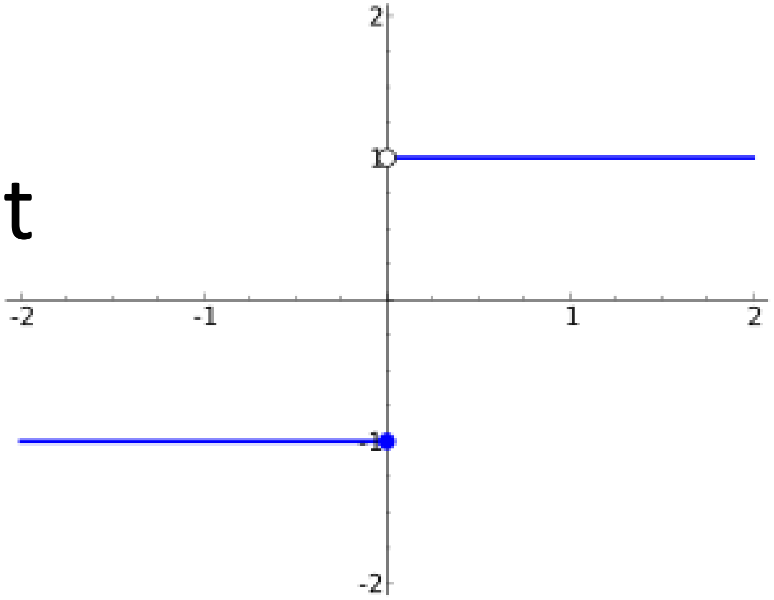
Removable Discontinuity (Hole)

- The function has a limit as $x \rightarrow 0$, and we can remove the discontinuity by setting f(0) equal to this limit.

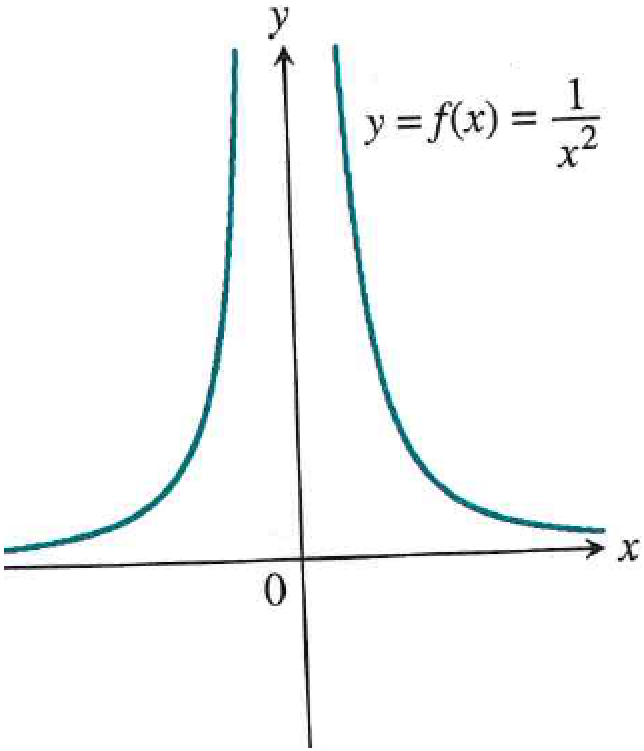


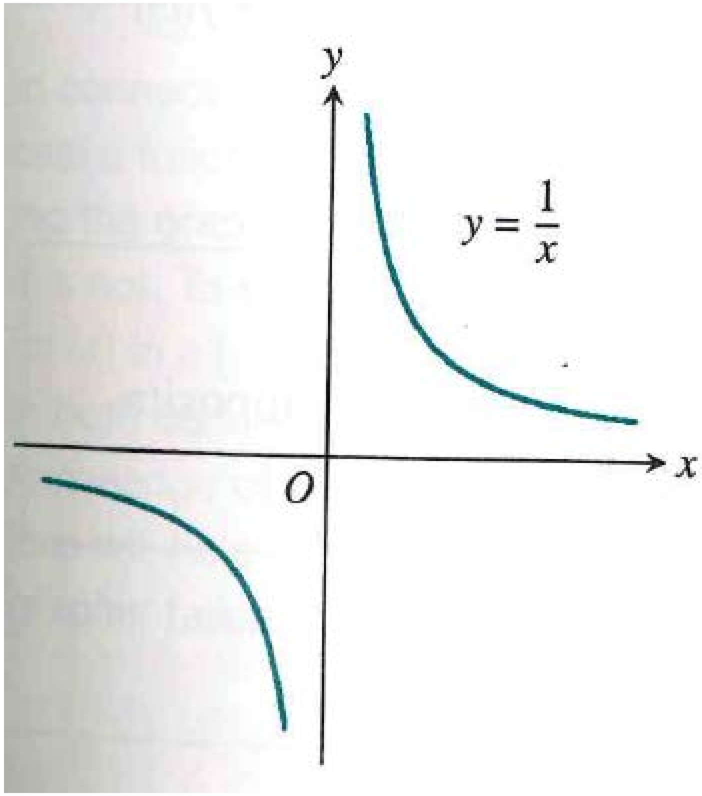
Jump Discontinuity

- The one sided limits exist, but have different values.
- $\lim_{x \to 0} f(x)$ does not exist and there is no way to improve the situation by changing f at 0.



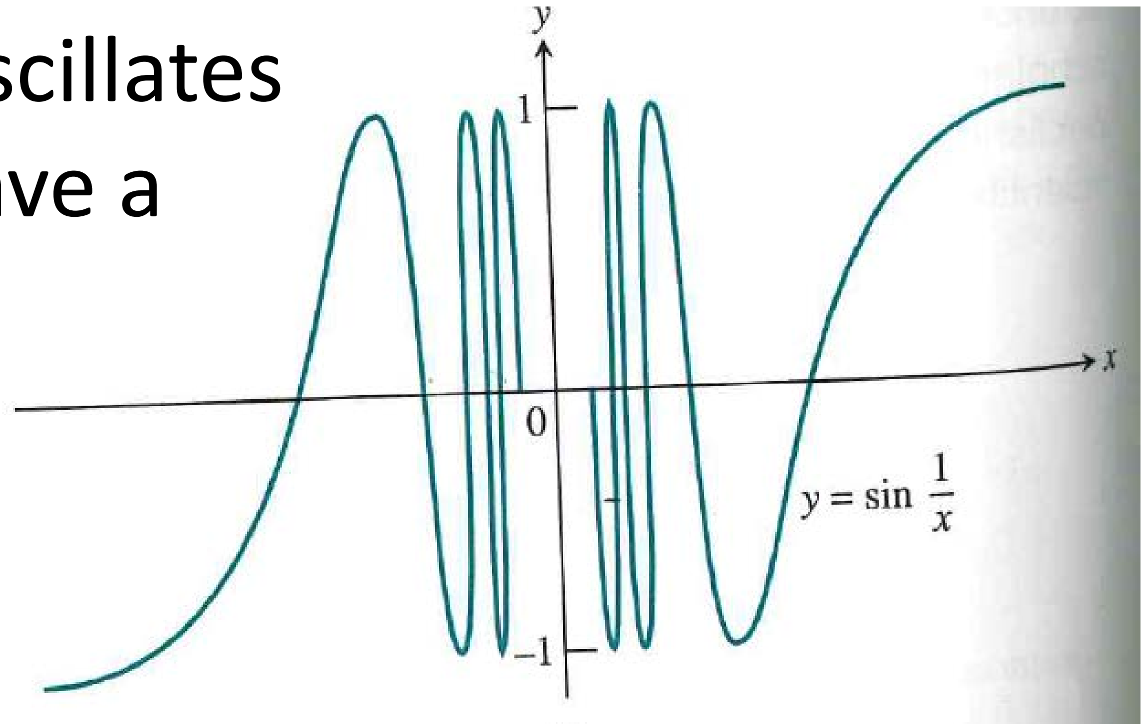
Infinite Discontinuity





Oscillating Discontinuity

- The function oscillates too much to have a limit as x→0.



(f)

Removing a discontinuity

- Let $f(x) = \frac{x^3-7x-6}{x^2-9}$
- Factor the denominator. What is the domain of *f*?
- Investigate the graph of *f* around x=3. Does *f* have a removable discontinuity?
- How should *f* be defined at x=3 to remove the discontinuity?
- Show that (x-3) is a factor of the numerator of *f*, and remove all common factors. Now compute the limit as x→3 of the reduced form of *f*.
- Show that the **extended function** $g(x) = \begin{cases} \frac{x^3-7x-6}{x^2-9}, & x \neq 3 \\ \frac{10}{3}, & x = 3 \end{cases}$ is continuous at x=3.

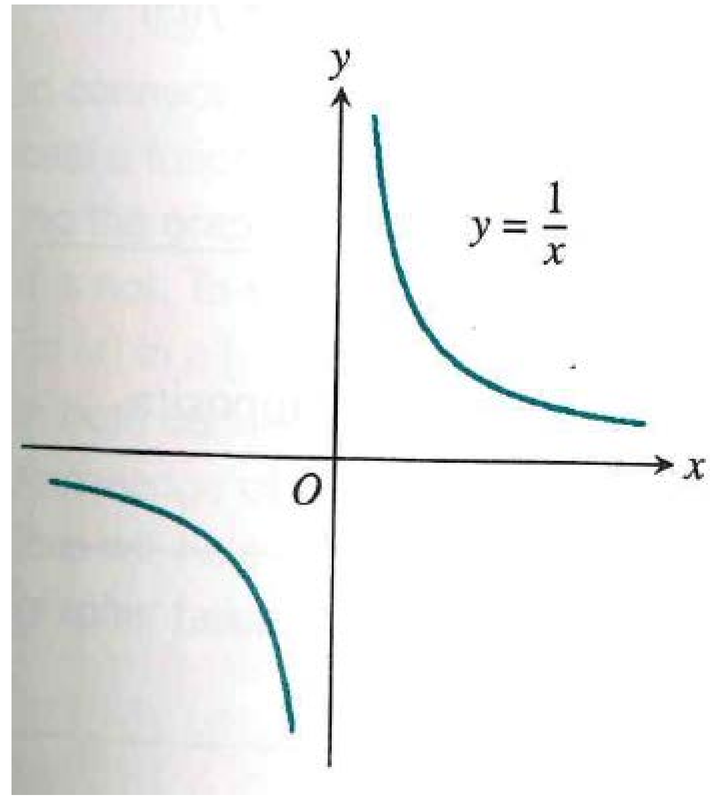
Vocab

- A function is **continuous on an interval** iff it is continuous at every point of the interval.
- A **continuous function** is one that is continuous at every point *of its domain*.

- Note: a continuous function does not have to be continuous on every interval. Example…

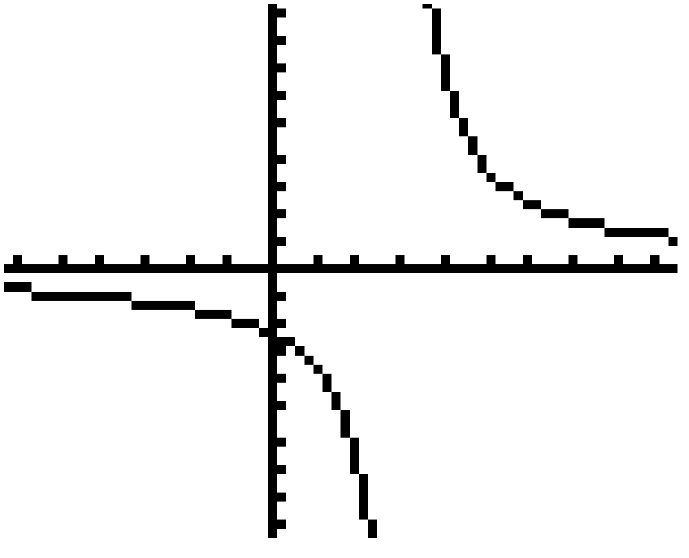
Is this a continuous function?

- What is the domain?
- Is the function continuous at every point *of its domain?*
- This is a continuous function because it is continuous at every point of it domain.
- It has a point of discontinuity at x=0 b/c it is not defined there.



Is this a continuous function?

- EX: The function y = $\frac{7}{x-3}$ has a domain of x ≠ 3.
- *Even though it's not continuous everywhere, it is continuous (meaning "continuous on it's domain")!*
- We can say it "has a point of discontinuity" at x = 3, because it is not defined there.



Properties of Continuous Functions

- Polynomial functions *f* are continuous at every real number *c* because $\lim_{x \to c} f(x) = f(c)$
- Rational functions are continuous at every point *of their domains*. They have points of discontinuity at the zeros of their denominators.
- The absolute value function *y=|x|* is continuous at every real number.
- The exponential, log, trig, and radical functions are continuous at every point of their domains.

Properties of Continuous Functions

Theorem 6 **Properties of Continuous Functions**

If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

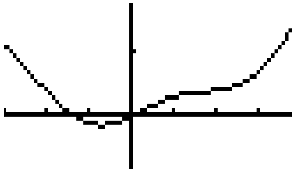
1. *Sums:*	$f + g$
2. *Differences:*	$f - g$
3. *Products:*	$f \cdot g$
4. *Constant multiples:*	$k \cdot f$, for any number k
5. *Quotients:*	f/g, provided $g(c) \neq 0$

Theorem 7 **Composite of Continuous Functions**

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

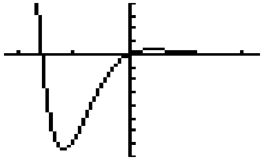
Examples of Combined Functions

- $f \pm g$ EX: $h(x) = \sin(x) + x^2$



- $f * g$ EX: $g(y) = \sin(y) * y^2$

- $k f$ for any scalar, k   EX: $r(x) = 9\sin(x)$
- $\frac{f}{g}$ provided $g(c) \neq 0$ EX: $s(t) = \frac{\sin(t)}{e^x}$



Conclusions (aka Take Home Message)

- You can remove “removable discontinuities” by defining an appropriate value at a given domain value.
- You cannot remove jump or infinite discontinuities from both sides. However, you may be able to remove a discontinuity from one side.

Conclusions (aka Take Home Message)

- A function is CONTINUOUS if it is continuous everywhere on its domain.
- It does NOT have to be continuous everywhere.
- If it is continuous only on some parts of the domain, we must state that clearly.
- And finally, we can only operate on a function over continuous pieces of the domain.

Conclusions (aka Take Home Message)

- For instance, the function y = $\frac{1}{x}$ has a discontinuity at x = 0, yet it is:
  - Continuous on its domain
  - Has a point of discontinuity at x = 0
  - Is *not* continuous on the interval [-1,1]
  - Is continuous on the interval (0, ∞) and again at (-∞ ,0)

Assignment

- pg 80 #1-3, 11-24