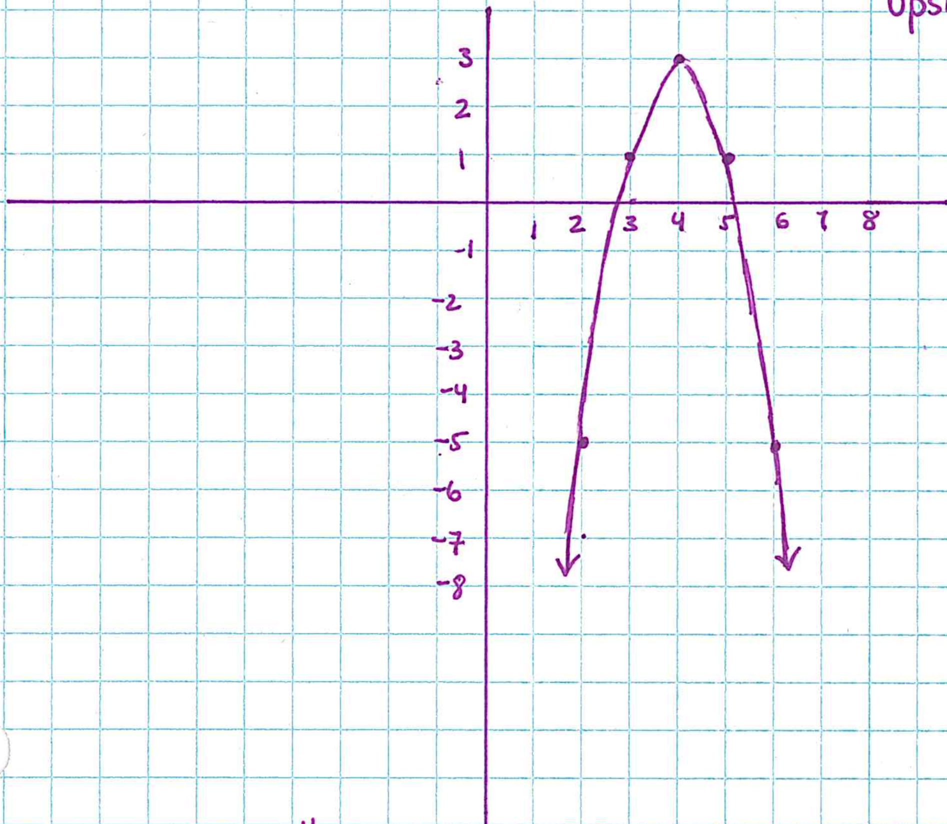
KEY

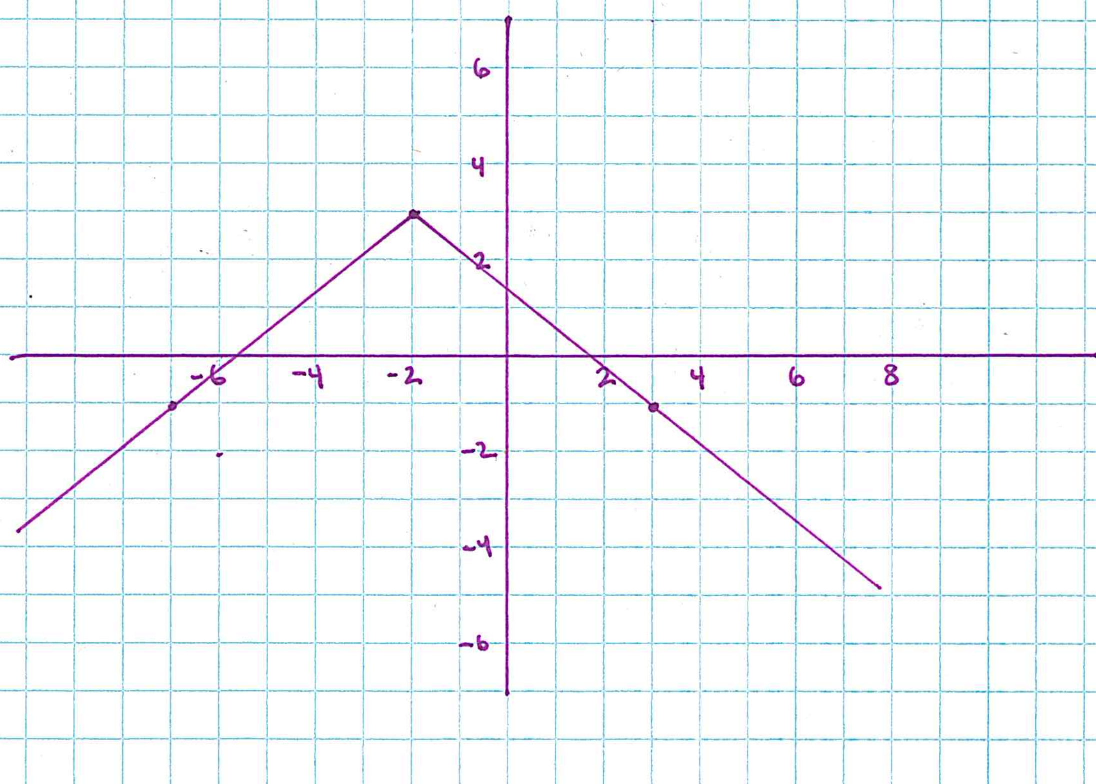
① $y = 3 - 2(4-x)^2$

$y = -2(-(x-4))^2 + 3$

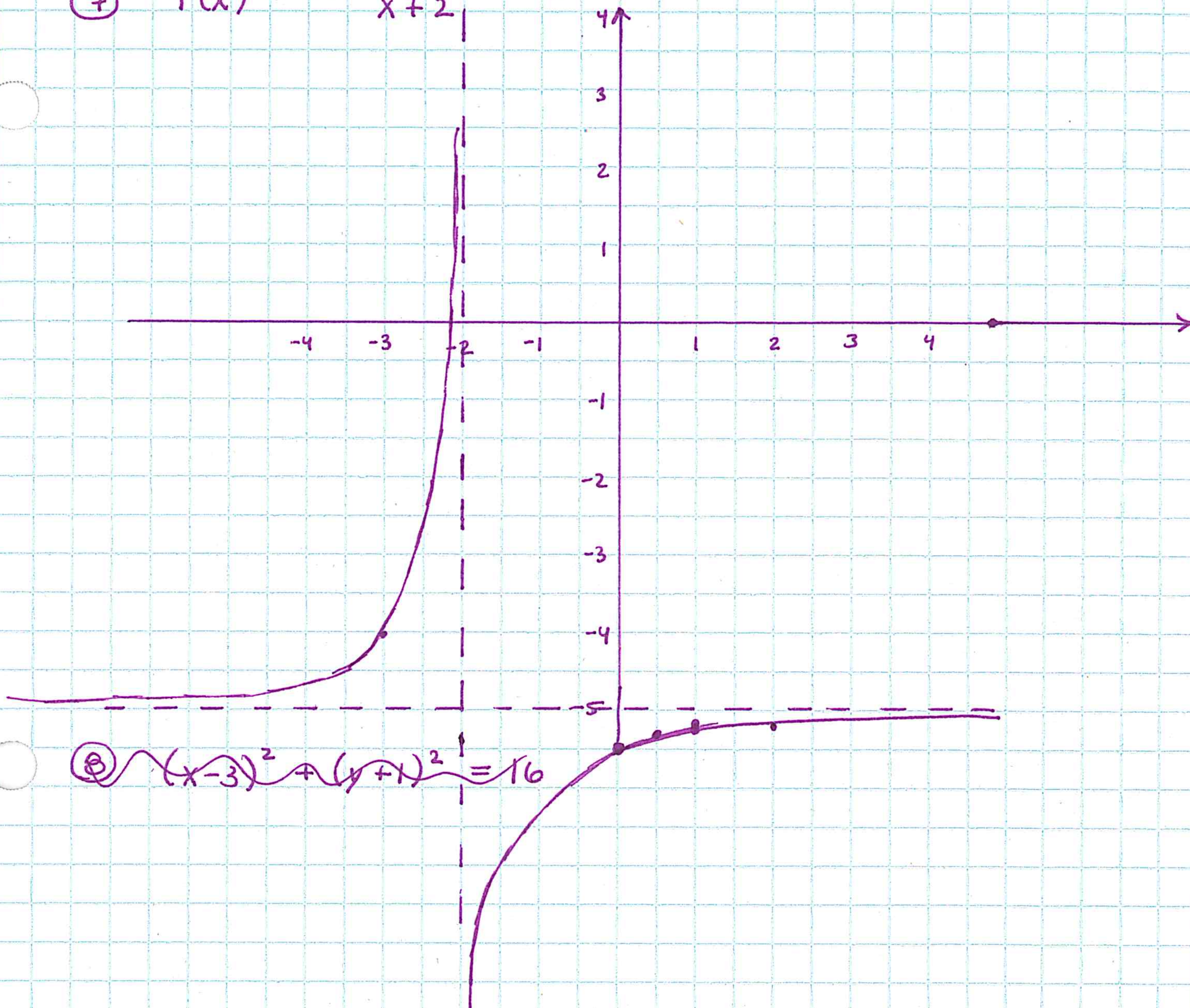
vertex at $(4,3)$
upside down



② $f(x) = -\frac{4}{5}|x+2| + 3$

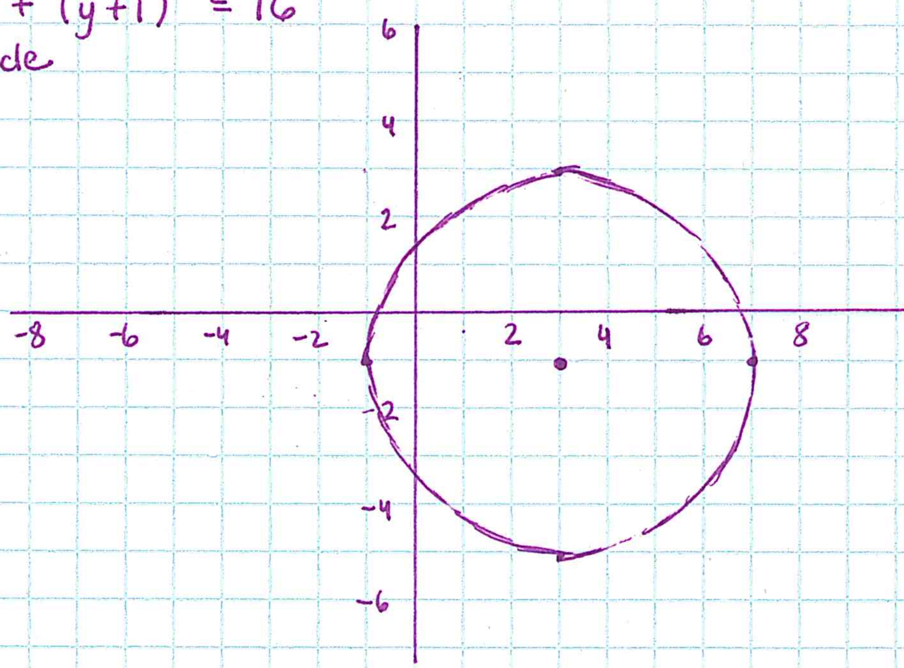


⑦ $f(x) = -\dfrac{1}{x+2} - 5$



⑧ $(x-3)^2 + (y+1)^2 = 16$

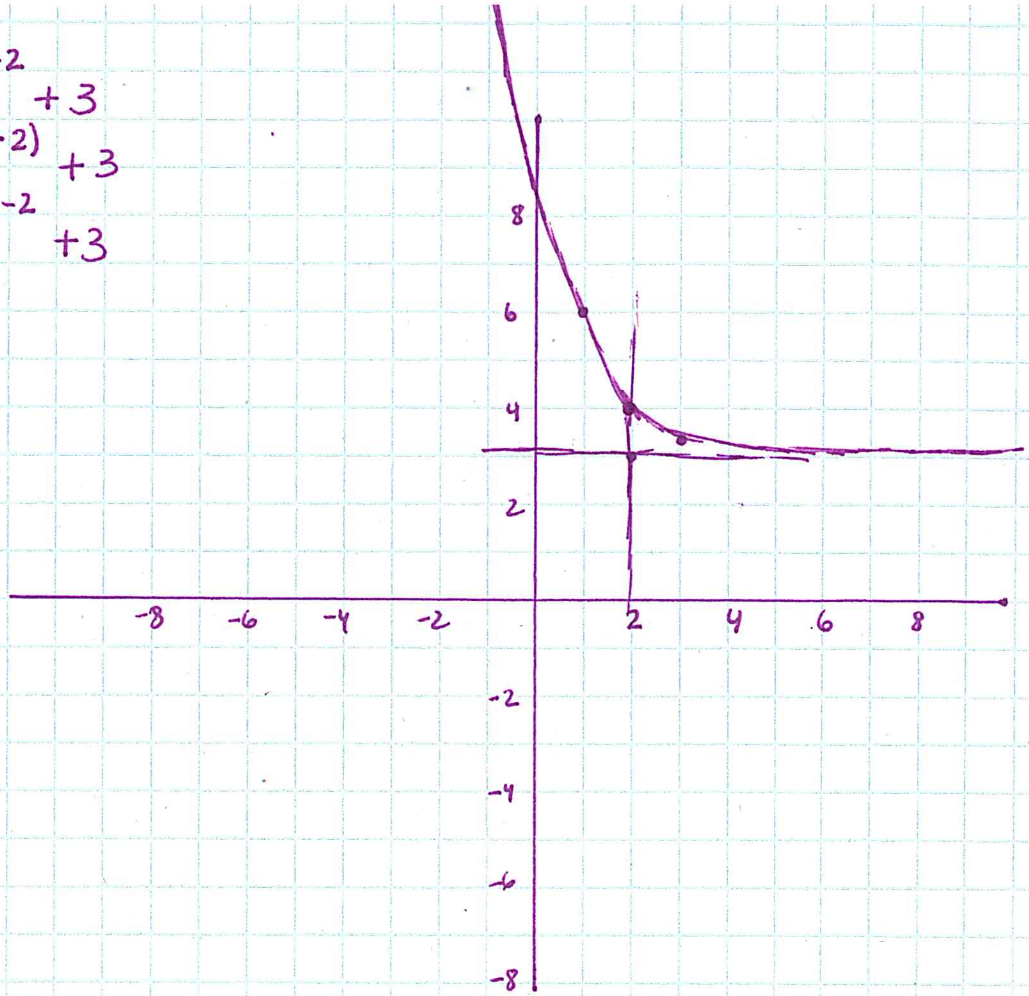
circle



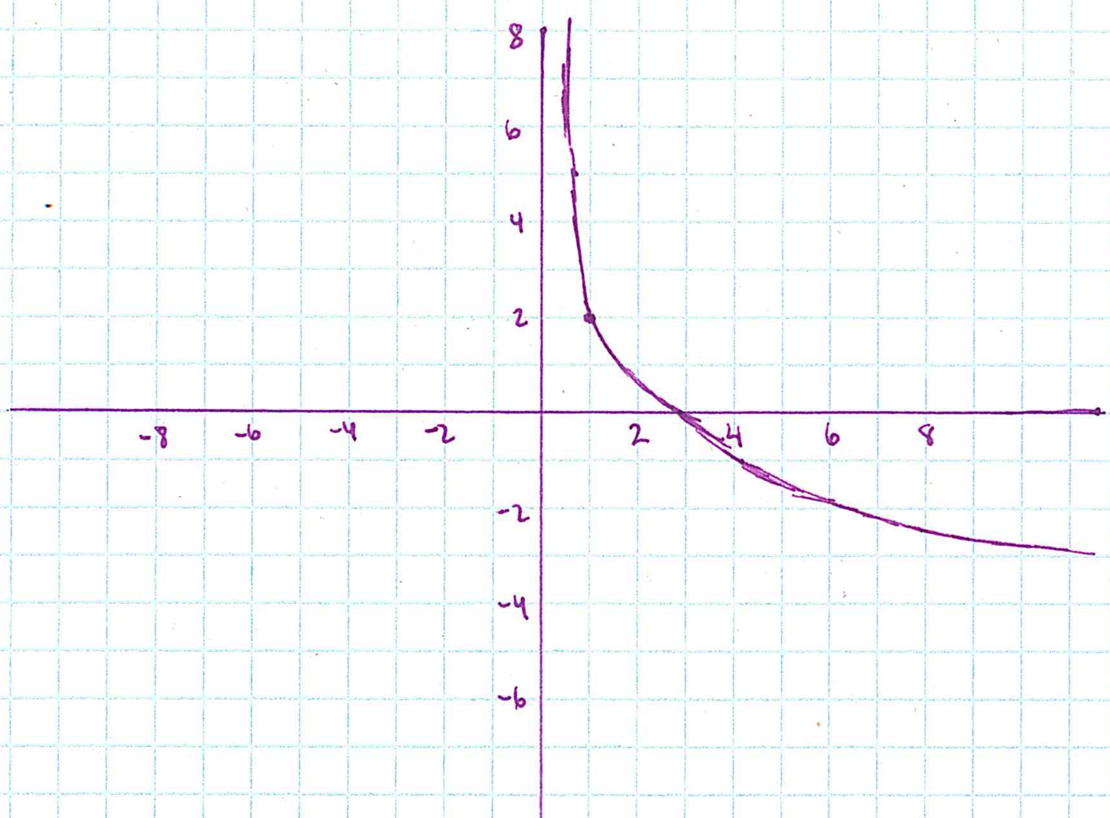
⑨ $f(x) = 3^{-x+2} + 3$

$f(x) = 3^{-(x-2)} + 3$

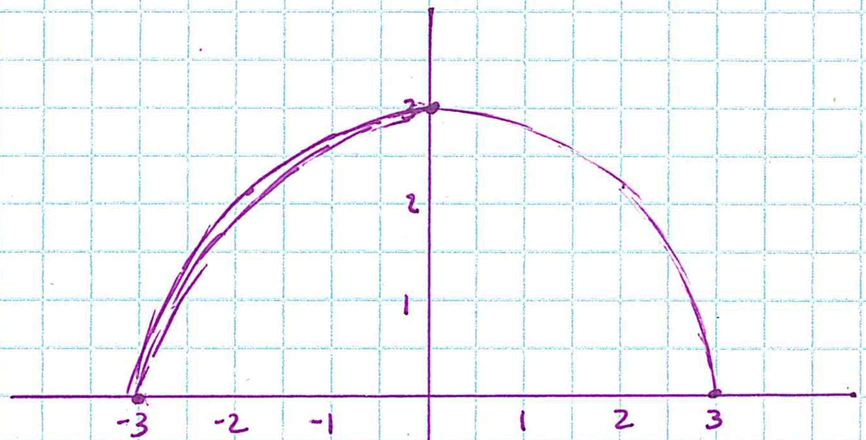
$= \left(\frac{1}{3}\right)^{x-2} + 3$



⑩ $y = -3\ln(x-1) + 2$



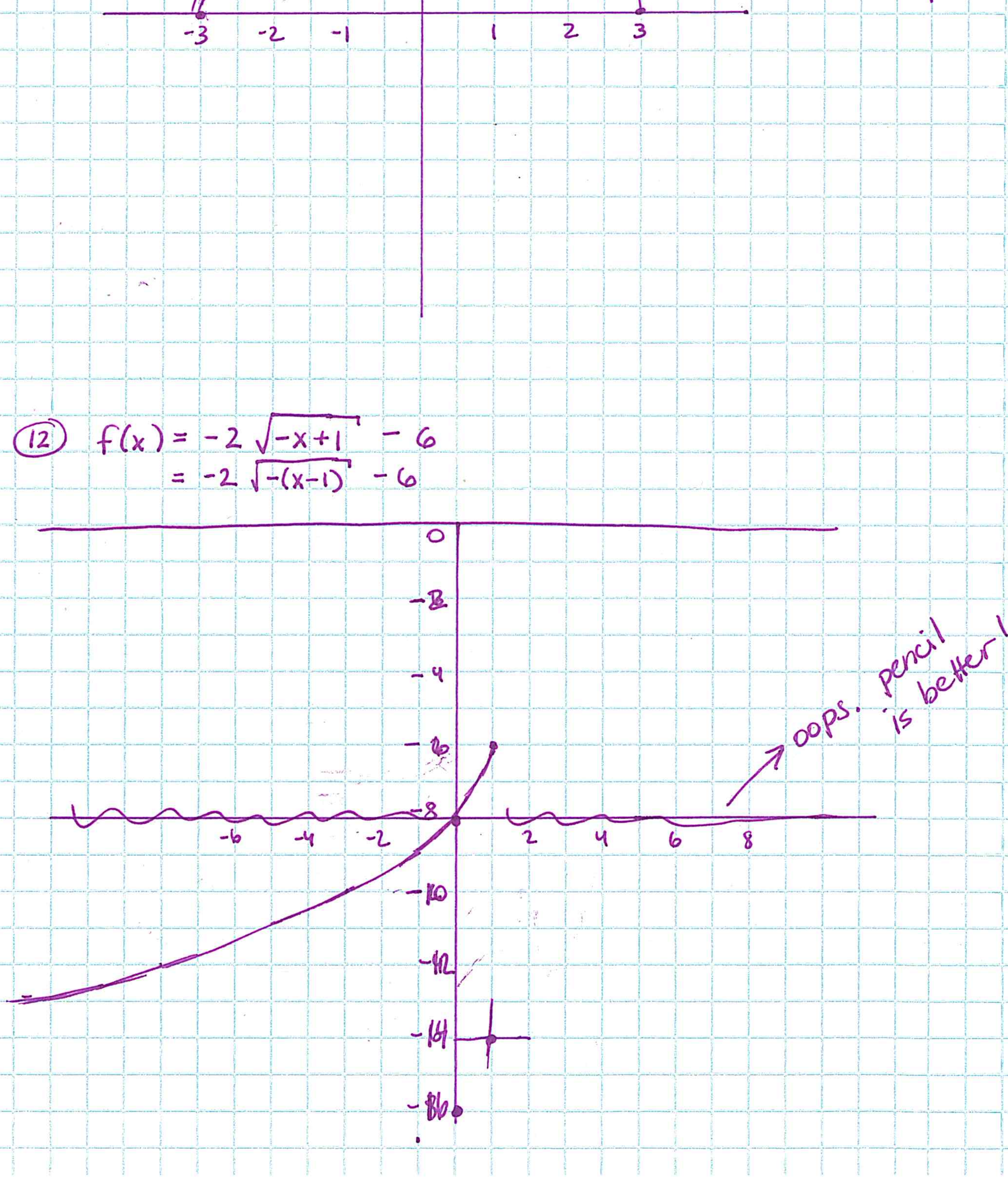
(11) $y = \sqrt{9-x^2}$



x	y
-3	0
-2	$\sqrt{5}$
0	3
2	$\sqrt{5}$
3	0

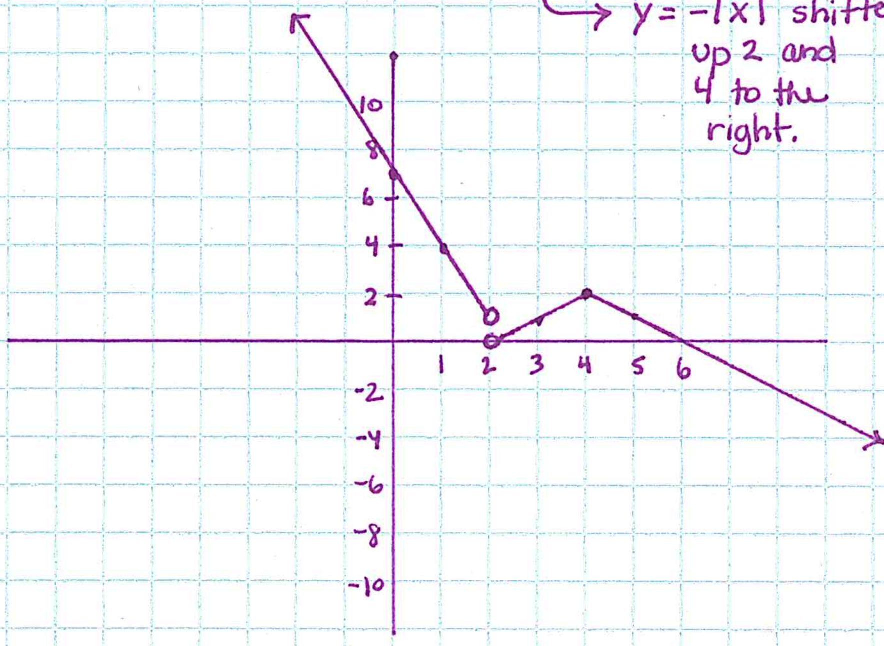
(12) $f(x) = -2\sqrt{-x+1} - 6$

$= -2\sqrt{-(x-1)} - 6$



(13) $f(x) = \begin{cases} -3x+7 & x<2 \\ -|x-4|+2 & x>2 \end{cases}$

$\to$ $y=-|x|$ shifted up 2 and 4 to the right.



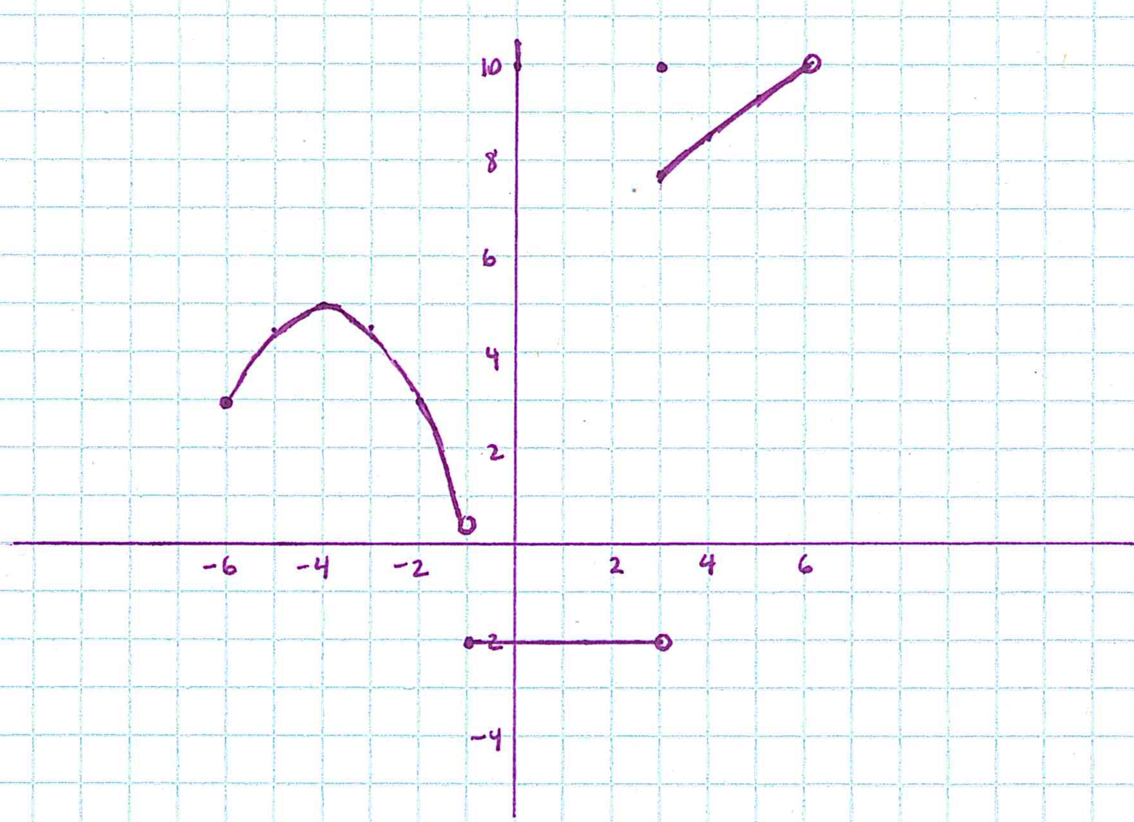
x	y
0	7
1	4

} use $y=-3x+7$ for these because $x<2$

Domain:
$x \in (-\infty, 2) \cup (2, \infty)$

Range:
$y \in (-\infty, \infty)$

(14) $g(x) = \begin{cases} -\frac{1}{2}(x+4)^2+5 & -6 \le x < -1 \\ -2 & -1 \le x < 3 \\ 4\sqrt{x+3} - 2 & 3 \le x < 6 \end{cases}$

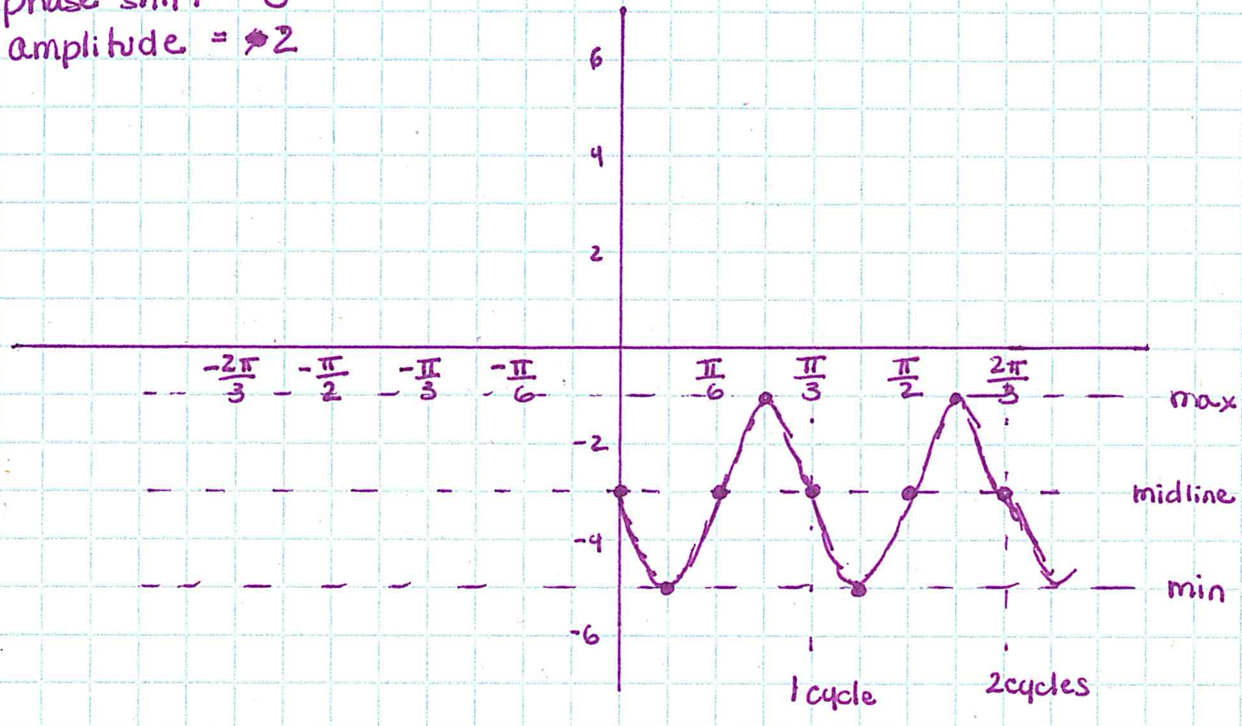


(15) $y = -2\sin 6x - 3$

period $= \frac{2\pi}{6} = \frac{\pi}{3}$

phase shift $= 0$

amplitude $= 2$

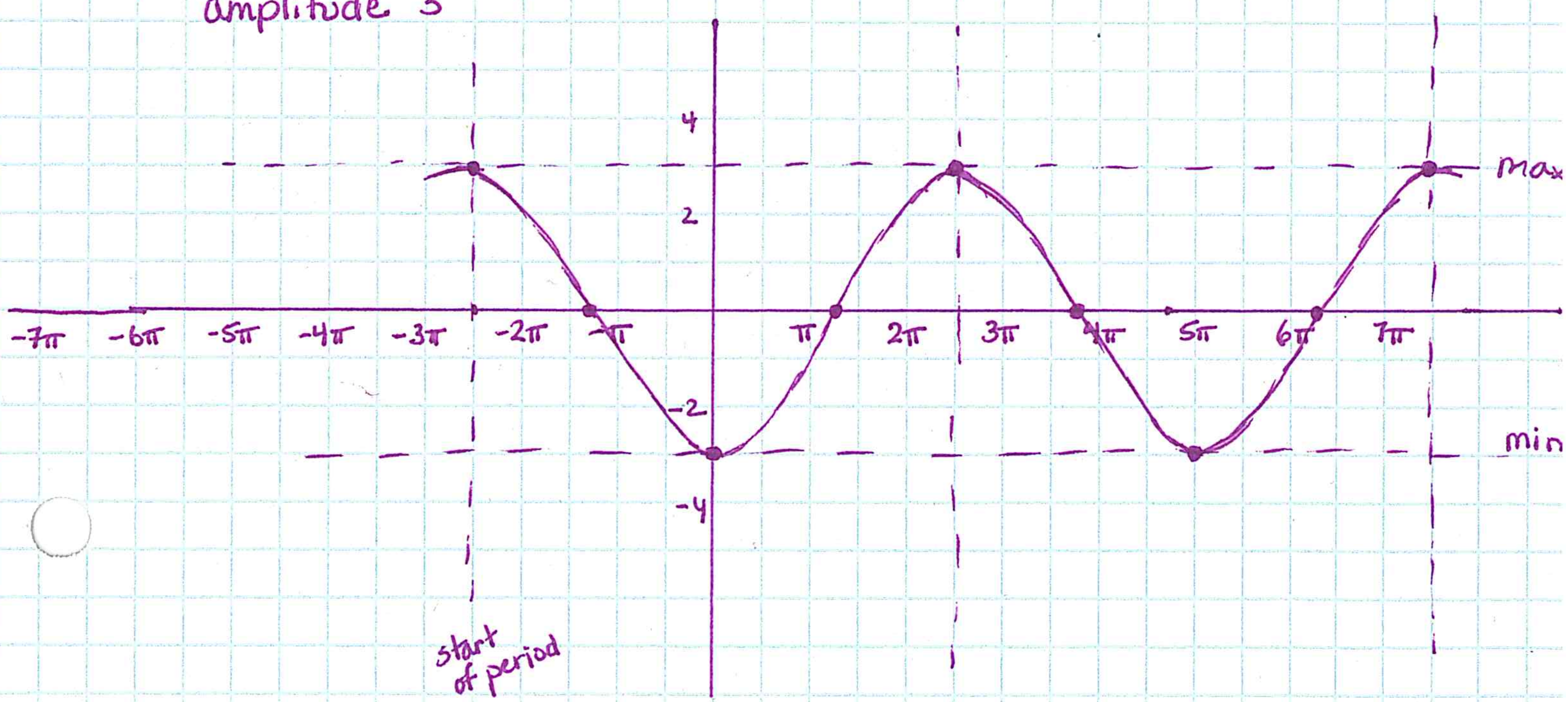


(16) $y = 3\cos\left(\frac{2x}{5} + \pi\right)$

period $= \frac{2\pi}{\frac{2}{5}} = 2\pi\left(\frac{5}{2}\right) = 5\pi$

phase shift $= -\frac{\pi}{\frac{2}{5}} = -\frac{5\pi}{2}$

amplitude 3



17 $y = -4 - \cos \pi x$

18 $y = \frac{1}{2} \csc\left(\frac{x}{2} - \frac{\pi}{3}\right) + 2$

19 $y = -2\sec\left(3x - \frac{3\pi}{4}\right)$

20 $y = \frac{2}{3}\tan\left(x + \frac{\pi}{4}\right) - 1$

21 $y = \cot\left(\frac{\pi x}{2} + \pi\right) + 3$

22 $y = \frac{1}{2}\cot\left(x - \frac{2\pi}{3}\right)$

(23) Find the domain & range

#2 $f(x) = -\frac{4}{5}|x+2| + 3$

domain: all real numbers $(-\infty, \infty)$

as $x \to -\infty$, $y \to -\infty$
as $x \to \infty$ $y \to -\infty$

range: notice that $-\frac{4}{5}|x+2|$ is always negative.
maximum y value will occur when $-\frac{4}{5}|x+2| = 0$
$y = 3$

$(-\infty, 3]$

#3. $y = -\frac{1}{3}\sqrt{2-x} + 1$

domain: $2 - x \geq 0$ $(-\infty, 2]$
$x \leq 2$

range: notice $-\frac{1}{3}\sqrt{2-x}$ is always negative & gets smaller ~~larger~~ for ~~larger~~ smaller values of x
max y value will be 1 (when $\sqrt{2-x} = 0$)

$(-\infty, 1]$

as $x \to -\infty$
$y \to -\infty$

#7. $f(x) = -\dfrac{1}{x+2} - 5$

domain $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

range as $x \to -\infty$ $y \to -5$
$x \to +\infty$ $y \to -5$
as $x \to -2.0001$ $y \to \infty$
as $x \to -1.9999$ $y \to -\infty$

$(-\infty, -5) \cup (-5, \infty)$

#11. $y = \sqrt{9 - x^2}$

domain: $9 - x^2 \geq 0$
$x \leq \pm 3$
$-3 \leq x \leq 3$

$[-3, 3]$

range: $\sqrt{9-x^2}$ is always positive ≥ 0
at $x = \pm 3$ $y = 0$ at $x = 0$ $y = 3$
$[0, 3]$

(24) Domain $(-\infty, \infty)$

Range $[-3, \infty)$

by reading graph

(25) Domain $(-\infty, 4) \cup (4, \infty)$

Range $[-2, \infty)$

(26) $f(x) = x^2 - 2x - 3$

if a function is odd $f(-x) = -f(x)$
if a function is even $f(-x) = f(x)$

$$f(-x) = (-x)^2 - 2(-x) - 3$$

$$= +x^2 + 2x - 3$$

$$\neq f(x)$$

$$\neq -f(x)$$

Neither even nor odd

(27)

$$f(x) = \frac{x^3}{8+x^2}$$

$$f(-x) = \frac{(-x)^3}{8+(-x)^2}$$

$$= \frac{-x^3}{8+x^2}$$

$$= -\left[\frac{x^3}{8+x^2}\right] = -f(x)$$

odd function

(28) $f(x) = e^{x^2}$

$$f(-x) = e^{(-x)^2}$$

$$= e^{x^2} = f(x)$$

even function

(29) $f(x) = \sin(x + \frac{\pi}{2})$

sin(x) is an odd function, but the $\frac{\pi}{2}$ phase shift changes things. Graph the function to see symmetry

(30) $y = \dfrac{x+3}{2x-7}$

change x and y
solve for y

$$x = \frac{y+3}{2y-7}$$

$$x(2y-7) = y+3$$

$$2xy - 7x = y + 3$$

$$\cancel{-y = 3 + 7x - 2xy}$$

$$2xy - y = 7x + 3$$

$$(2x-1)y = 7x+3$$

$$\boxed{y = \frac{7x+3}{2x-1}}$$

this is a function

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$y = 5 + 2\log_3(x+5)$

$x = 5 + 2\log_3(y+5)$

$\frac{x-5}{2} = \log_3(y+5)$

$3^{\left(\frac{x-5}{2}\right)} = y+5$

$\boxed{y = 3^{\frac{x-5}{2}} - 5}$

this is an exponential function

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$f(x) = \dfrac{2+5x}{5x-2}$

$g(x) = \sqrt{9-x^2}$

$h(x) = x^2-9$

(34) $(h \circ g)(x) = h\left(\sqrt{9-x^2}\right)$

$= \left(\sqrt{9-x^2}\right)^2 - 9$

$= -x^2 \qquad -3 \le x \le 3$

(35) $f(g(x)) = f\left(\sqrt{9-x^2}\right)$

$= \dfrac{2+5\left(\sqrt{9-x^2}\right)}{5\left(\sqrt{9-x^2}\right)-2}$

(36) $f(g(h(x)))$

(37)

(38)

39 $(5x^2z^6)^3(5x^2yz^{-2})^{-3} = \cancel{5^3}\,\cancel{x^6}\,z^{18} \cdot \cancel{5^{-3}}\,\cancel{x^{-6}}\,y^{-3}z^{6}$

$= \dfrac{z^{24}}{y^3}$

40 $\dfrac{2(x^{-3}y^4)^{-2}}{(2x^{-1}y^5z)^2} = \dfrac{2x^{-6}y^8}{2^2x^{-2}y^{10}z^2} = \boxed{\dfrac{1}{2x^4y^2z^2}}$

$2^{1-2}x^{-6-(-2)}y^{8-10}z^{-2}$

41 $3\ln x - \ln(x+3) + 2\ln y$

$\boxed{= \ln\left(\dfrac{x^3y^2}{x+3}\right)}$

42 $\log\left(\dfrac{(x-1)^3}{y^2z}\right) = \log(x-1)^3 - \log(y^2z)$

$= 3\log(x-1) - \log y^2 - \log z$

$\boxed{= 3\log(x-1) - 2\log y - \log z}$

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