p80 #1-3, 11-24

① $y = \frac{1}{(x+2)^2}$

since this is the quotient of polynomials, this function is continuous on its domain. The only points of discontinuity are those point(s) where the function is not defined — where the denominator is 0.

$x+2=0$

$x=-2$ is an infinite discontinuity.

$\lim_{x\to -2^-} f(x) = \infty$ $\lim_{x\to -2^+} f(x) = \infty$

② $y = \frac{x+1}{x^2-4x+3} = \frac{x+1}{(x-3)(x-1)}$

The function is continuous b/c it is a quotient of polynomials which are continuous.

The only discontinuities are where the function is not defined

$(x-3)(x-1) = 0$

$x=3$ and $x=1$ are infinite discontinuities.

$\lim_{x\to 3^-} f(x) = -\infty$ $\lim_{x\to 3^+} f(x) = \infty$

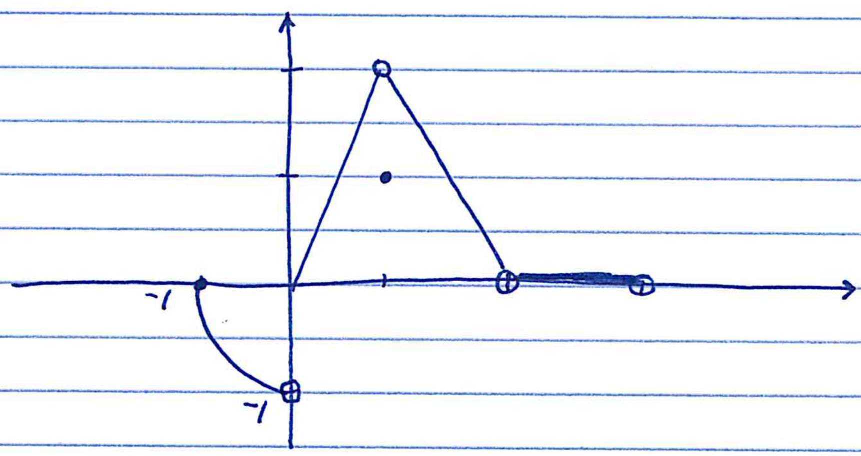
$\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to 1^+} f(x) = \infty$

③ $y = \frac{1}{x^2+1}$

continuous function on its domain x^2+1 will never $= 0$, so there are no points of discontinuity.

(11)

$$f(x) = \begin{cases} x^2 - 1 & -1 \le x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x < 2 \\ 0 & 2 < x < 3 \end{cases}$$



(11) (a) $f(-1) = (-1)^2 - 1 = 0$ $\quad$ $f(-1)$ exists

(b) $\lim_{x \to -1^+} f(x)$ exists $\quad$ $\lim_{x \to -1^+} f(x) = 0$

(c) $\lim_{x \to -1^+} f(x) = 0 = f(-1)$ $\quad$ yes

(d) yes, f is continuous at $x = -1$ (since -1 is a left endpoint, only the one sided limit must exist and be equal to $f(-1)$)

(12) (a) $f(1) = 1$ exists

(b) $\lim_{x \to 1} f(x) = 2$

(c) $\lim_{x \to 1} f(x) \neq f(1)$

(d) f is not continuous at $x = 1$

(13) a) f is not defined at 2
b) f is not continous at 2, since 2 is not in the domain.

(14) f is continuous for $x \in [-1,0) \cup (0,1) \cup (1,2) \cup (2,3)$
<u>or</u> f is continuous everywhere in $[-1,3)$ except $x = 0, 1, 2$

(15) since $\lim_{x \to 2} f(x) = 0$ we need $f(2) = 0$ for the function to be continuous at $x = 2$

(16) since $\lim_{x \to 1} f(x) = 2$ we need $f(1) = 2$ for the function to be continuous at 2.

(17) We cannot extend the function to be continuous at $x=0$ because there is a jump discontinuity at $x=0$. The left hand limit and the right hand limit are not the same.

(18) We can extend the function at $x=3$ by giving it the value $f(3) = 0$, since $\lim_{x \to 3^-} f(x) = 0$.

Since $x=3$ is a right endpoint only the left hand limit needs to exist.

(19) $f(x) = \begin{cases} 3-x & x < 2 \\ \frac{x}{2}+1 & x > 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = \frac{2}{2} + 1 = 2$

there is a jump discontinuity at $x = 2$
it is not removable.

(20) $f(x) = \begin{cases} 3-x & x<2 \\ 2 & x=2 \\ \frac{x}{2} & x>2 \end{cases}$

$\lim_{x\to 2^-} f(x) = 3-2 = 1$

$\lim_{x\to 2^+} f(x) = \frac{2}{2} = 1$

$\lim_{x\to 2} f(x) = 1$

$f(2) = 2$

there is a hole at $x=2$. It is removable and can be removed by assigning $f(2)=1$.

(21) $f(x) = \begin{cases} \frac{1}{x-1} & x<1 \\ x^3-2x+5 & x \geq 1 \end{cases}$

$\lim_{x\to 1^-} f(x) = \frac{1}{1-1}$ can't substitute

as $x \to 1^-$ (from the left) the denominator is a very small negative number

$\lim_{x\to 1^-} f(x) = -\infty$ infinite discontinuity at $x=1$

not removable

(22) $f(x) = \begin{cases} 1-x^2 & x \neq -1 \\ 2 & x=-1 \end{cases}$

$\lim_{x\to -1^-} f(x) = 1-(-1)^2 = 0$

$\lim_{x\to -1^+} f(x) = 1-(-1)^2 = 0$

$\lim_{x\to -1} f(x) = 0$

$f(-1) = 2$ there is a hole at $x=-1$

it can be removed by defining $f(-1)=0$

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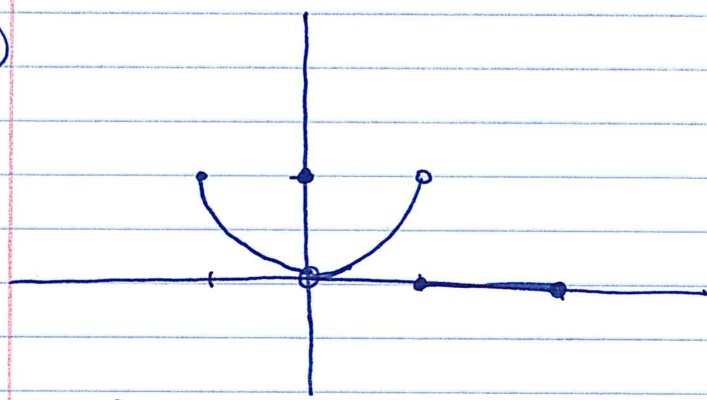

(a) $f(x)$ is discontinuous for all points not in the domain, and at $x = 0$ (hole) and $x = 1$ (jump discontinuity).

(b) The hole at $x = 0$ can be removed by defining $f(0) = 0$ because $\lim_{x \to 0} f(x) = 0$

The discontinuity at $x = 1$ cannot be removed because the left hand limit is different than the right hand limit.

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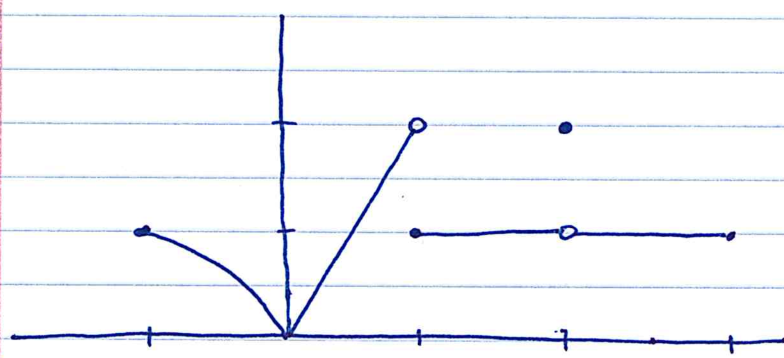

(a) The function is continuous on $[-1, 3]$ except at $x = 1$ and $x = 2$

(b) The discontinuity at $x = 1$ cannot be removed b/c the R.H. limit is different from the L.H. limit

The discontinuity at $x = 2$ can be removed by defining $f(2) = 1$ because $\lim_{x \to 2} f(x) = 1$