P80 # 25 – 30, 35 – 38, 41

25

$$f(x) = \frac{x^2-9}{x+3} = \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}$$ hole at $x = -3$

The extended function $\boxed{y = x - 3}$ is continuous at $x = -3$

or

$$\boxed{g(x) = \begin{cases} \frac{x^2-9}{x+3} & x \neq -3 \\ -6 & x = -3 \end{cases}}$$

is continuous at $x = -3$

26

$$f(x) = \frac{x^3-1}{x^2-1} = \frac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}$$ hole at $x = 1$

The extended function $y = \frac{x^2+x+1}{x+1}$ behaves like $f(x)$ and is continuous at $x = 1$

or

$$g(x) = \begin{cases} \frac{x^3-1}{x^2-1} & x \neq 1 \\ \frac{3}{2} & x = 1 \end{cases}$$

is continuous at $x = 1$

27

$f(x) = \frac{\sin x}{x}$, $x = 0$

there is a hole at $x = 0$, need to find a value of $f(0)$ to plug the hole

$$f(0) = \lim_{x \to 0} f(x)$$

$$f(0) = \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\boxed{g(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}}$$

(28) $f(x) = \dfrac{\sin 4x}{x}$, $x=0$

There is a hole at $x=0$. Need to find a value of $f(0)$ to plug the hole

$$f(0) = \lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$$

$$= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot 4$$

$$= \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \lim_{x\to 0} 4$$

$$= 1 \cdot 4$$

$$= \underline{\underline{4}}$$

$$\boxed{g(x) = \begin{cases} \dfrac{\sin 4x}{x} & x \neq 0 \\ 4 & x = 0 \end{cases}}$$

(29) $f(x) = \dfrac{x-4}{\sqrt{x}-2}$, $x=4$

$$f(x) = \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}$$

The function $\boxed{\sqrt{x}+2}$ behaves like $f(x)$ and is continuous at $x=4$

or

$$g(x) = \begin{cases} \dfrac{x-4}{\sqrt{x}-2} & x \neq 4 \\ 4 & x = 4 \end{cases}$$

is also an extended function that is continuous at $x=4$.

(30) $f(x) = \dfrac{x^3-4x^2-11x+30}{x^2-4}$

$x=2$

$$\begin{array}{r|l} & x^2-2x-15 \\ \hline x-2 & x^3-4x^2-11x+30 \\ & x^3-2x^2 \\ \hline & -2x^2-11x \\ & -2x^2+4x \\ \hline & -15x+30 \\ & -15x+30 \\ \hline & 0 \end{array}$$

$$= \frac{\cancel{(x-2)}(x^2-2x-15)}{(x+2)\cancel{(x-2)}}$$

$$= \frac{x^2-2x-15}{x+2}$$

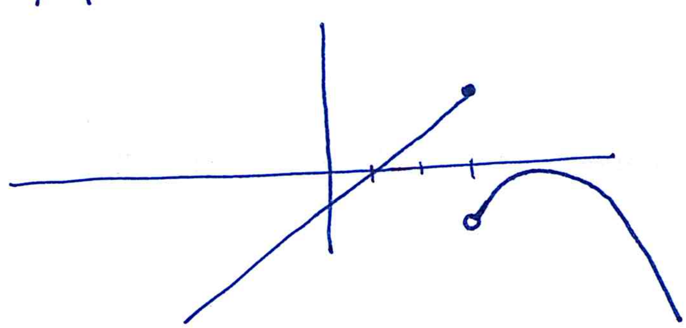
$\boxed{y = \dfrac{x^2-2x-15}{x+2}}$ is an extended function that behaves like $f(x)$ and is continuous at $x=2$

or

$$g(x) = \begin{cases} \dfrac{x^3-4x^2-11x+30}{x^2-4} & x \neq 2 \\ -\dfrac{15}{4} & x = 2 \end{cases}$$

(35) $f(3)$ exists, but $\lim\limits_{x \to 3} f(x)$ does not.

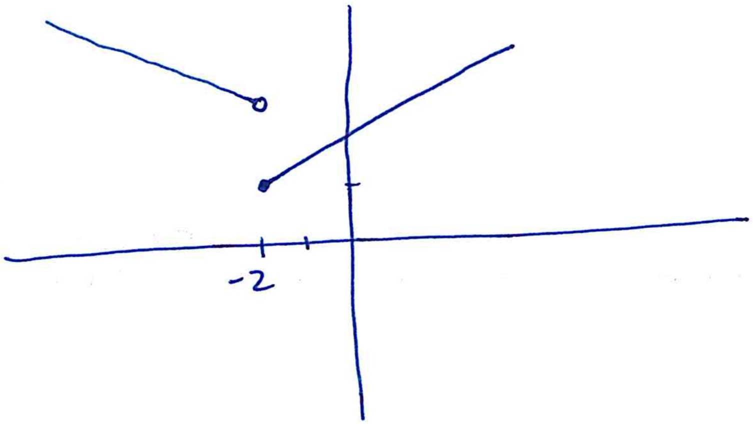
Many possible answers!



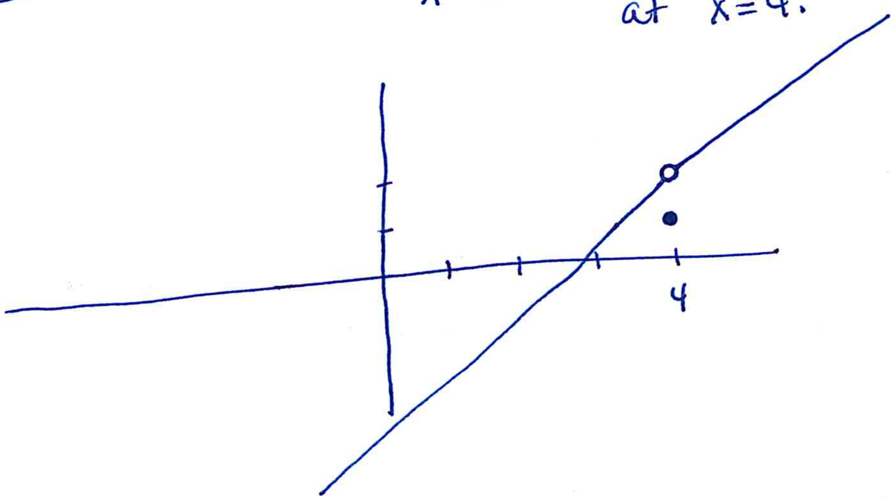
36 $f(-2)$ exists

$\lim_{x \to -2^+} f(x) = f(-2)$ but $\lim_{x \to -2} f(x)$ does not exist.

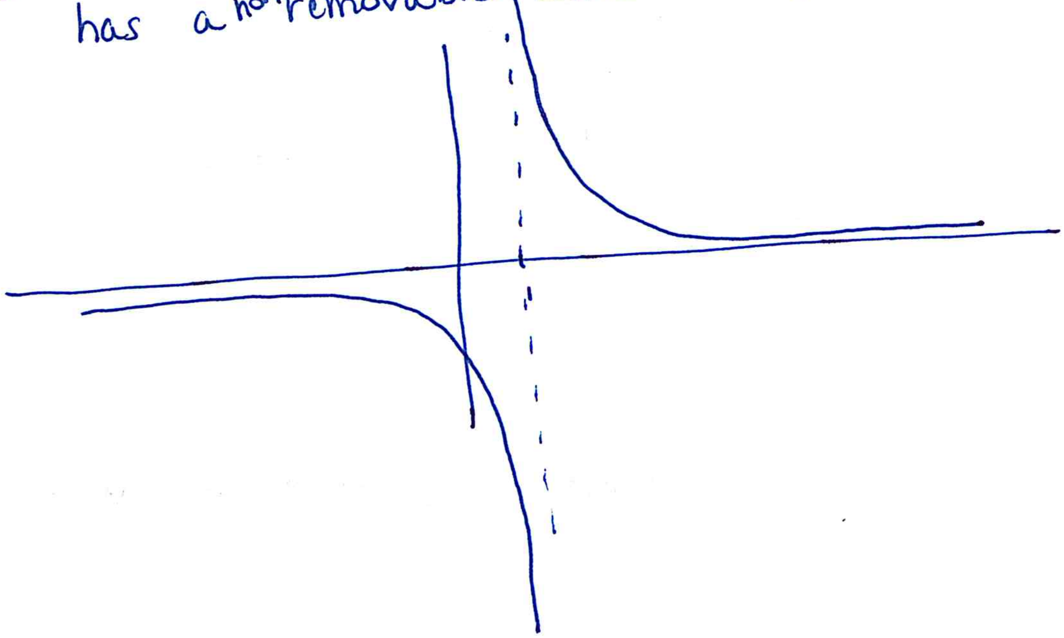
Many answers!



37 $f(4)$ exists, $\lim_{x \to 4} f(x)$ exists but f is not continuous at $x=4$.



38 $f(x)$ is continuous for all x except $x=1$, where f has a non removable discontinuity.



(41) $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$

For x to be continuous at all values of x

$$f(3) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$f(3) = (3)^2 - 1 = 2a(3)$$

$$f(3) = 9 - 1 = 6a$$

$$\frac{8}{6} = a$$

$$\boxed{\frac{4}{3} = a}$$

$$f(x) = \begin{cases} x^2 - 1 & x < 3 \\ \frac{8}{3}x & x \geq 3 \end{cases}$$