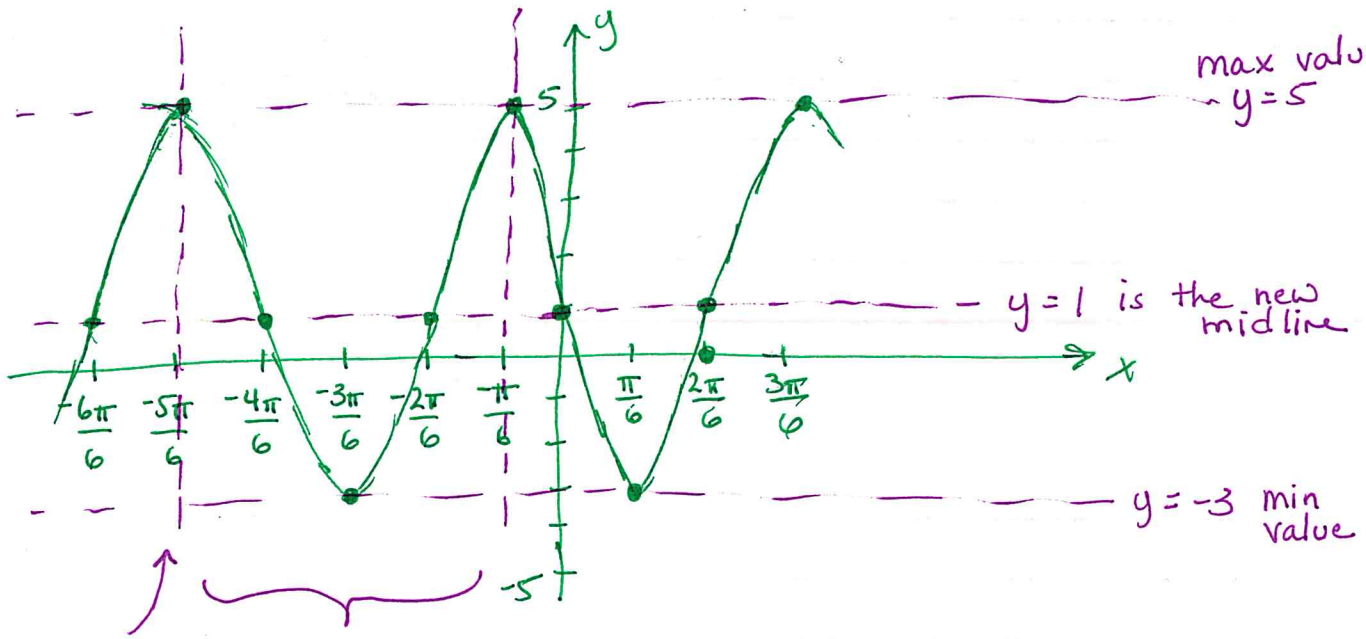
$y = a\cos(b(x - \frac{c}{b})) + d$

(5) $y = 4\cos(3(\theta + \frac{5\pi}{6})) + 1$ → Note that this form of the equation gives you the phase shift

phase shift $= -\frac{5\pi}{6}$

period $= \frac{2\pi}{3}$



phase shift (start of period)

one period

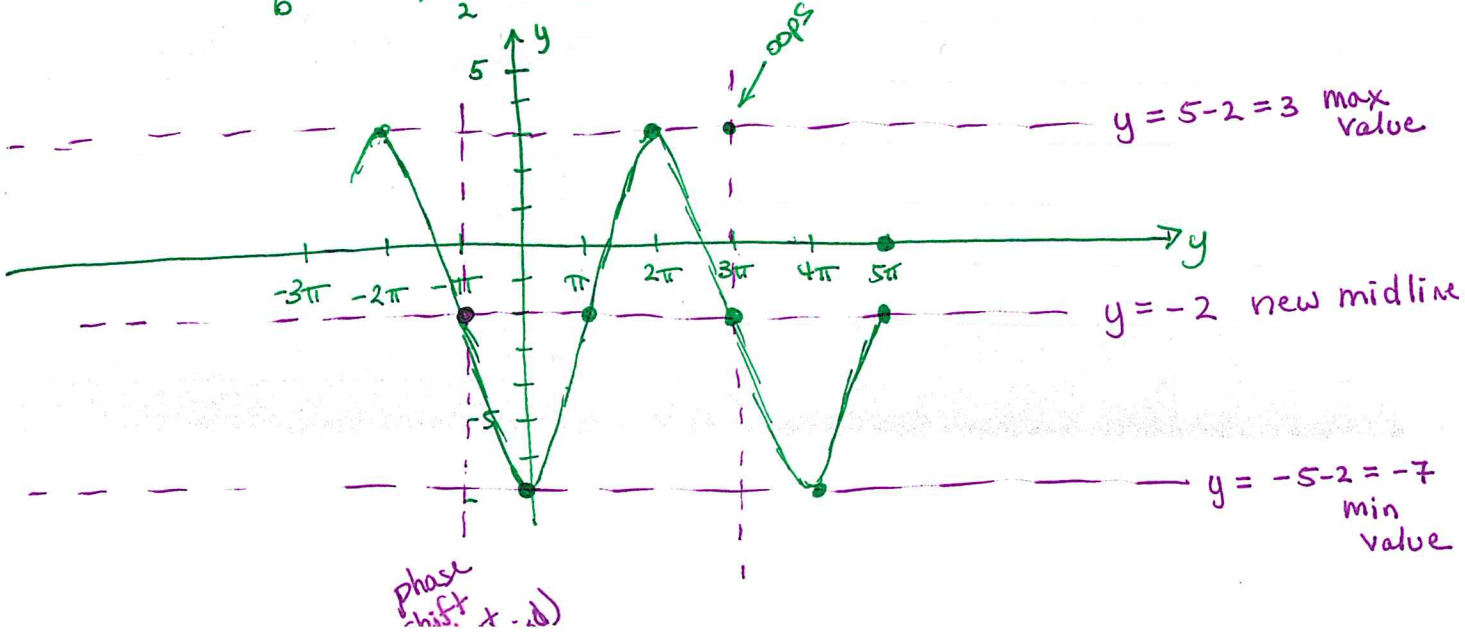
$y = a\sin(bx - c) + d$

Notice this is in a slightly different form than the one above

(6) $y = -5\sin(\frac{\theta}{2} + \frac{\pi}{2}) - 2$

phase shift $= \frac{c}{b} = \frac{\pi}{2} / \frac{1}{2} = -\pi$

period $= \frac{2\pi}{b} = 2\pi / \frac{1}{2} = 4\pi$



phase shift (+, -)

⑦ $y = \tan\left(\frac{\theta}{4}\right)$ $\quad \tan\left(\frac{\theta}{4}\right) = \frac{\sin\left(\frac{\theta}{4}\right)}{\cos\left(\frac{\theta}{4}\right)}$

asymptotes at $\frac{\theta}{4} = -\frac{\pi}{2}$

$\theta = -2\pi$

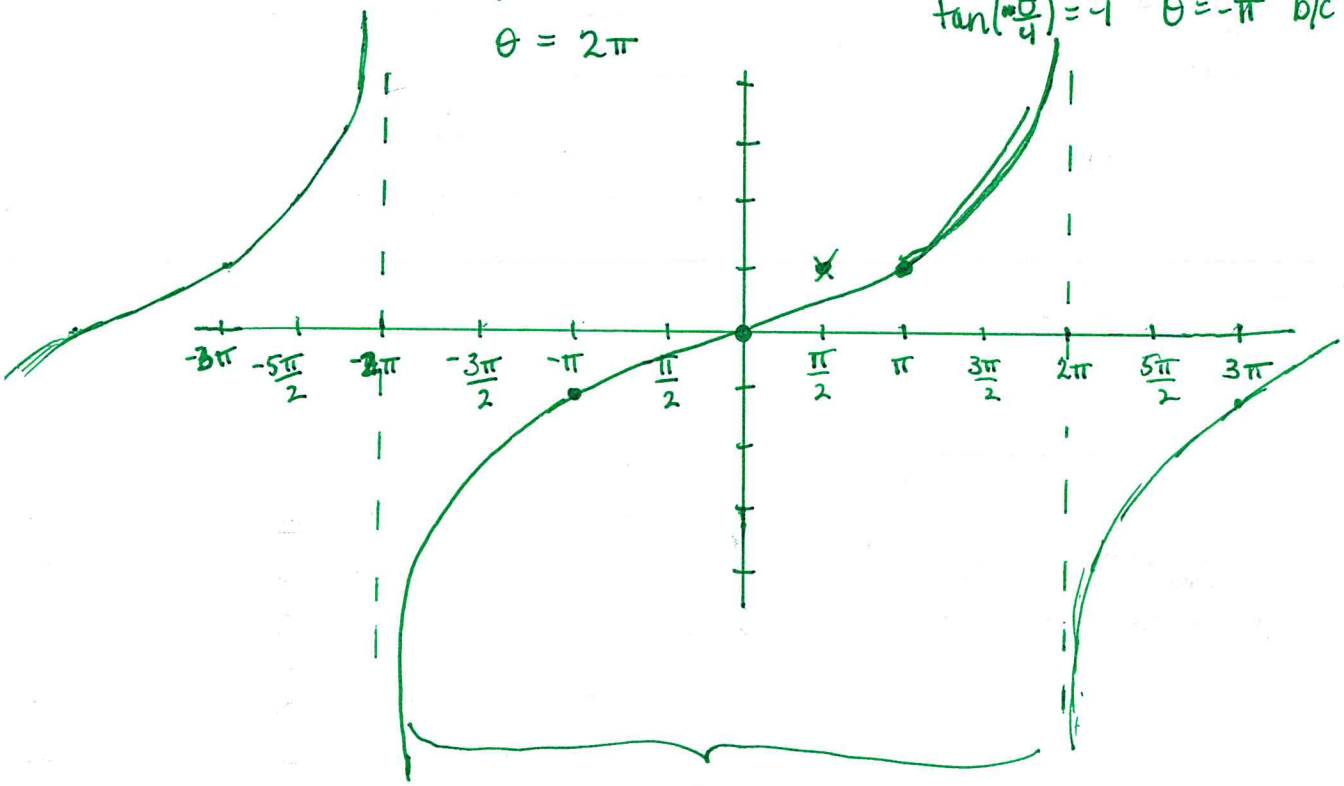
and $\frac{\theta}{4} = +\frac{\pi}{2}$

$\theta = 2\pi$

$\tan\left(\frac{\theta}{4}\right) = 0$ when $\sin\left(\frac{\theta}{4}\right) = 0$ which is at $\theta = 0$

$\tan\left(\frac{\theta}{4}\right) = 1$ $\quad \theta = \pi$ b/c $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$

$\tan\left(-\frac{\theta}{4}\right) = -1$ $\quad \theta = -\pi$ b/c $\sin\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$



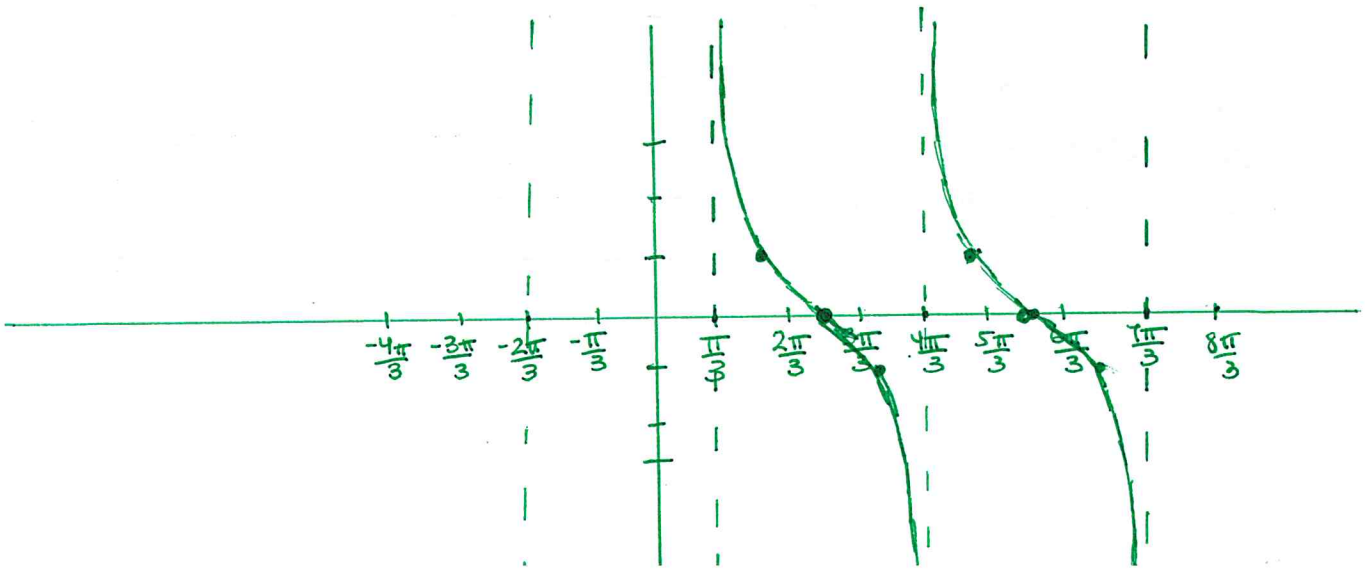
one period

⑧ $y = \cot\left(\theta - \frac{\pi}{3}\right)$ $\quad \cot\left(\theta - \frac{\pi}{3}\right) = \frac{\cos\left(\theta - \frac{\pi}{3}\right)}{\sin\left(\theta - \frac{\pi}{3}\right)}$

asymptotes at $\theta - \frac{\pi}{3} = 0 \Rightarrow \theta = \frac{\pi}{3}$

and $\theta - \frac{\pi}{3} = \pi \Rightarrow \theta = \frac{4\pi}{3}$

x	y
$\frac{7\pi}{12}$	1
$\frac{10\pi}{12}$	-1



(9) $y = -3\cot(2\theta)$

asymptotes at $2\theta = 0 \Rightarrow \theta = 0$

$2\theta = \pi \Rightarrow \theta = \frac{\pi}{2}$

$y = -3\cot(2\theta) = -3\left(\frac{\cos(2\theta)}{\sin(2\theta)}\right)$

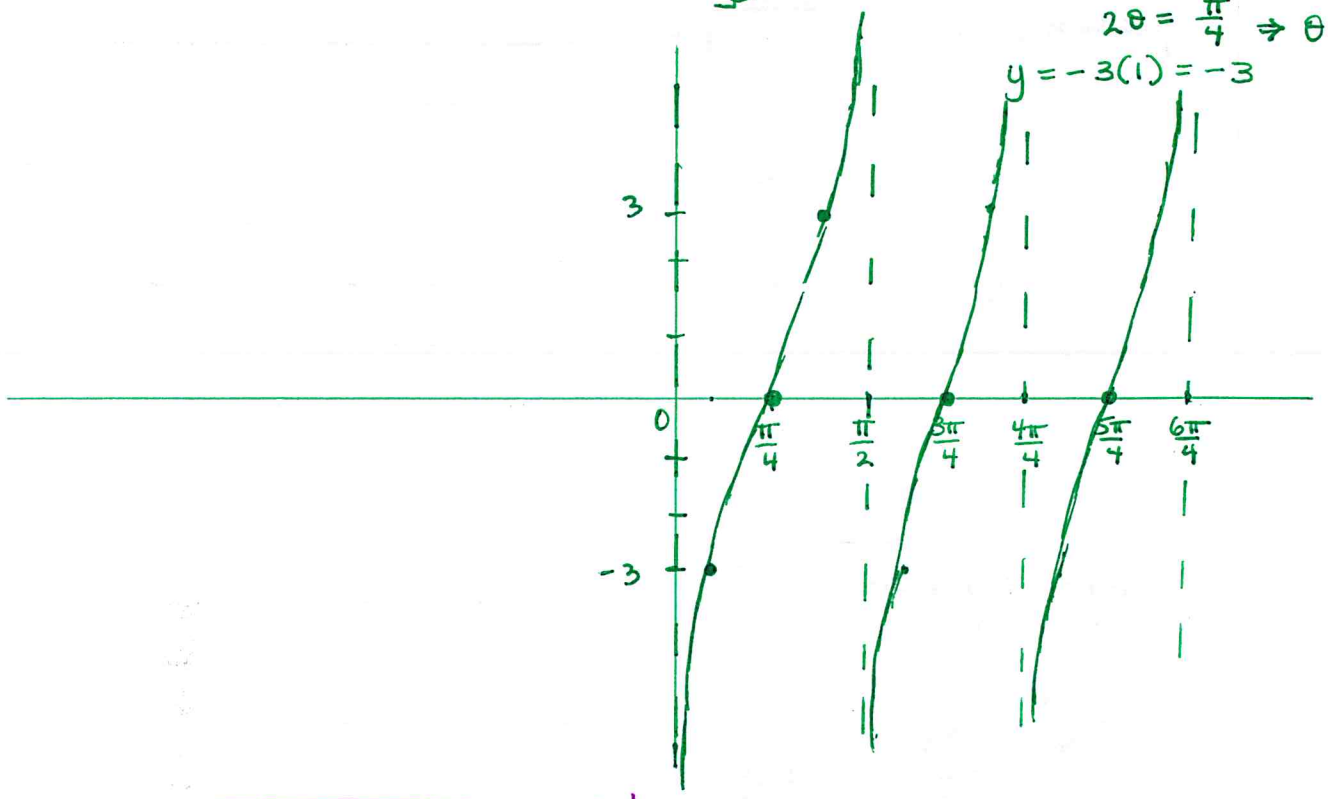
$y = 0$ when $\cos(2\theta) = 0$ $\quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\cot(2\theta) = 1$ when $\cos 2\theta = \sin 2\theta$

$2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8}$

$y = -3(1) = -3$



(10) $y = 2\sec(\pi(\theta+3)) - 4$ graphed in purple

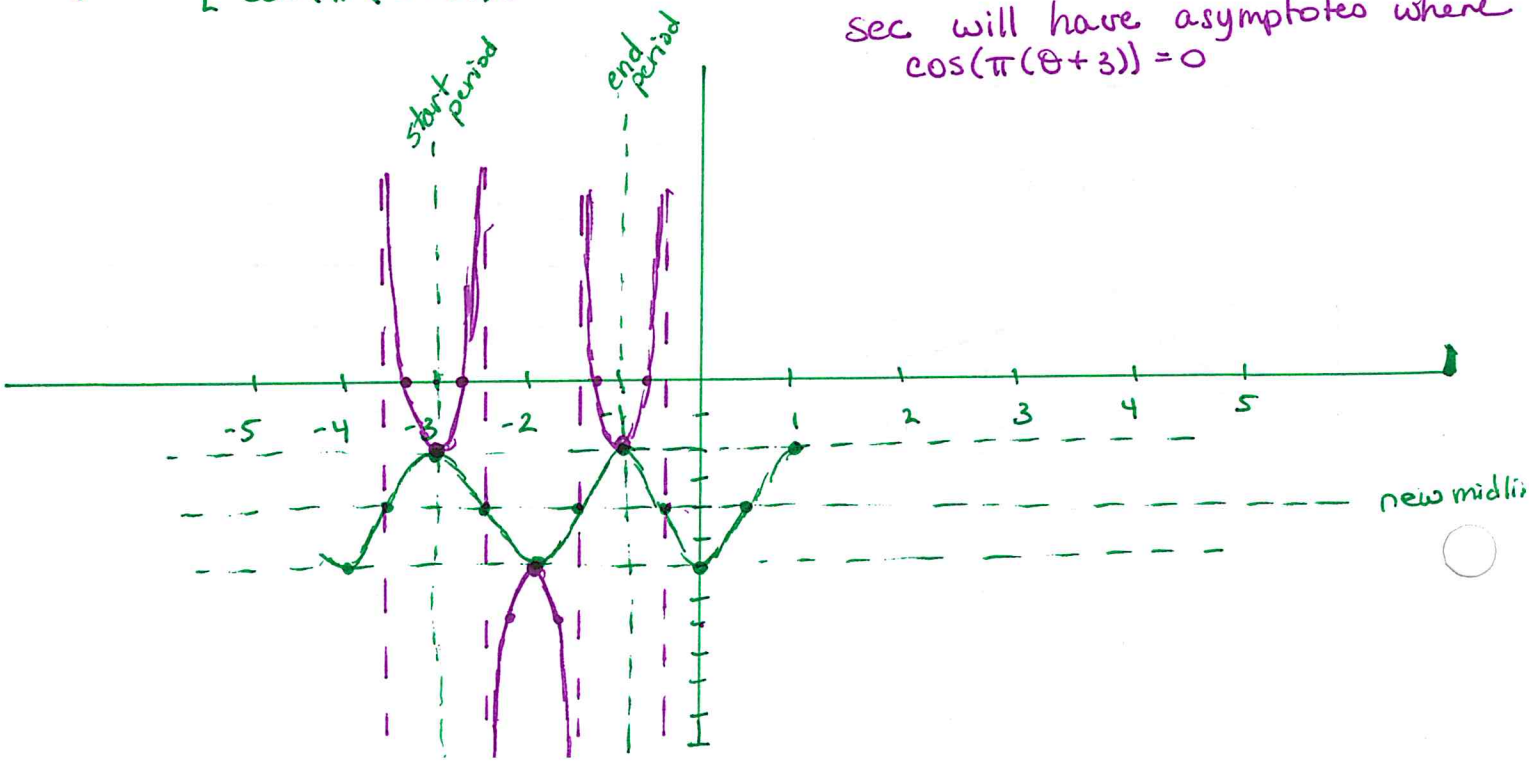
$y = 2\left[\frac{1}{\cos(\pi(\theta+3))}\right] - 4$

First graph $y = 2\cos(\pi(\theta+3)) - 4$

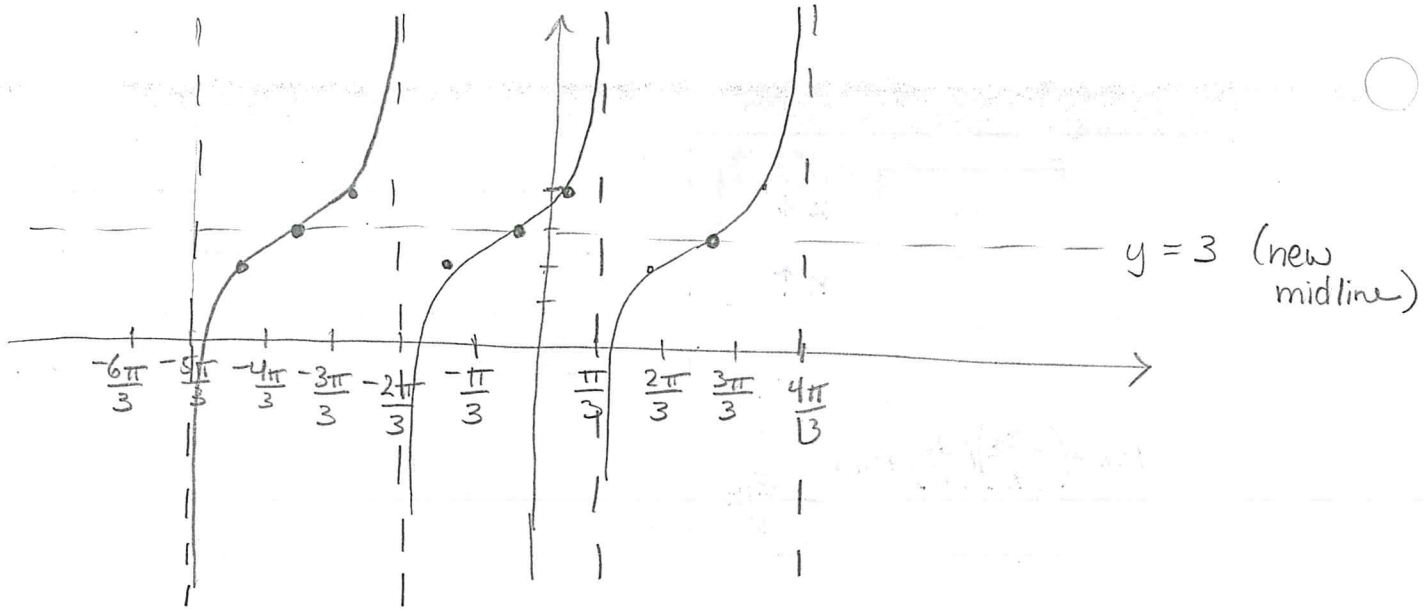
phase shift $= -3$

period $= \frac{2\pi}{\pi} = 2$

sec. will have asymptotes where $\cos(\pi(\theta+3)) = 0$



(11) $y = \tan\left(\theta + \frac{7\pi}{6}\right) + 3$



L.A. $\theta + \frac{7\pi}{6} = -\frac{\pi}{2}$

$\theta = -\frac{\pi}{2} - \frac{7\pi}{6}$

$= -\frac{3\pi}{6} - \frac{7\pi}{6}$

$= -\frac{10\pi}{6} = -\frac{5\pi}{3}$

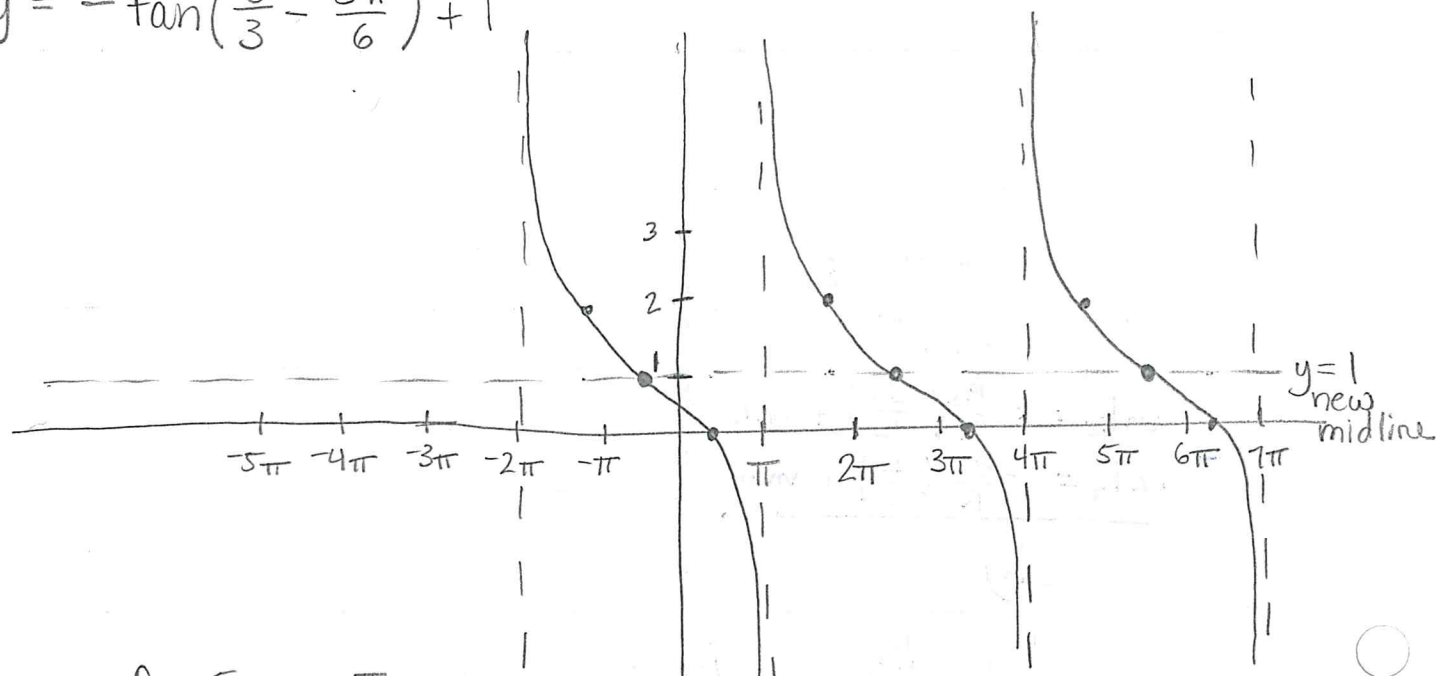
R.A. $\theta + \frac{7\pi}{6} = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} - \frac{7\pi}{6}$

$\theta = \frac{3\pi}{6} - \frac{7\pi}{6}$

$\theta = -\frac{4\pi}{6} = -\frac{2\pi}{3}$

(12) $y = -\tan\left(\frac{\theta}{3} - \frac{5\pi}{6}\right) + 1$



L.A. $\frac{\theta}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$

$\frac{\theta}{3} = -\frac{3\pi}{6} + \frac{5\pi}{6}$

$\theta = \pi$

R.A. $\frac{\theta}{3} - \frac{5\pi}{6} = \frac{\pi}{2}$

$\frac{\theta}{3} = \frac{3\pi}{6} + \frac{5\pi}{6}$

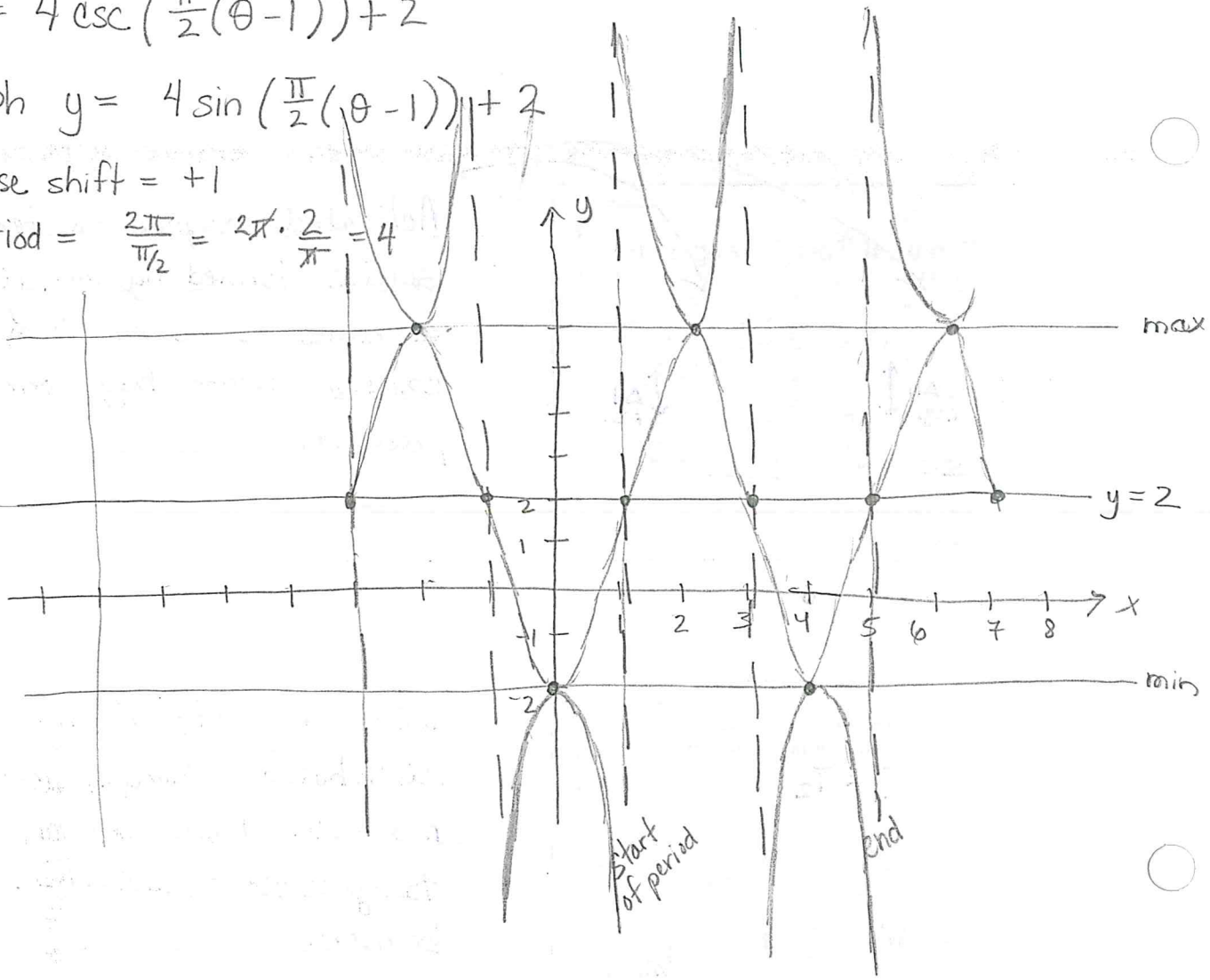
$\frac{\theta}{3} = \frac{8\pi}{6}$ $\theta = 4\pi$

(13) $y = 4\csc\left(\frac{\pi}{2}(\theta - 1)\right) + 2$

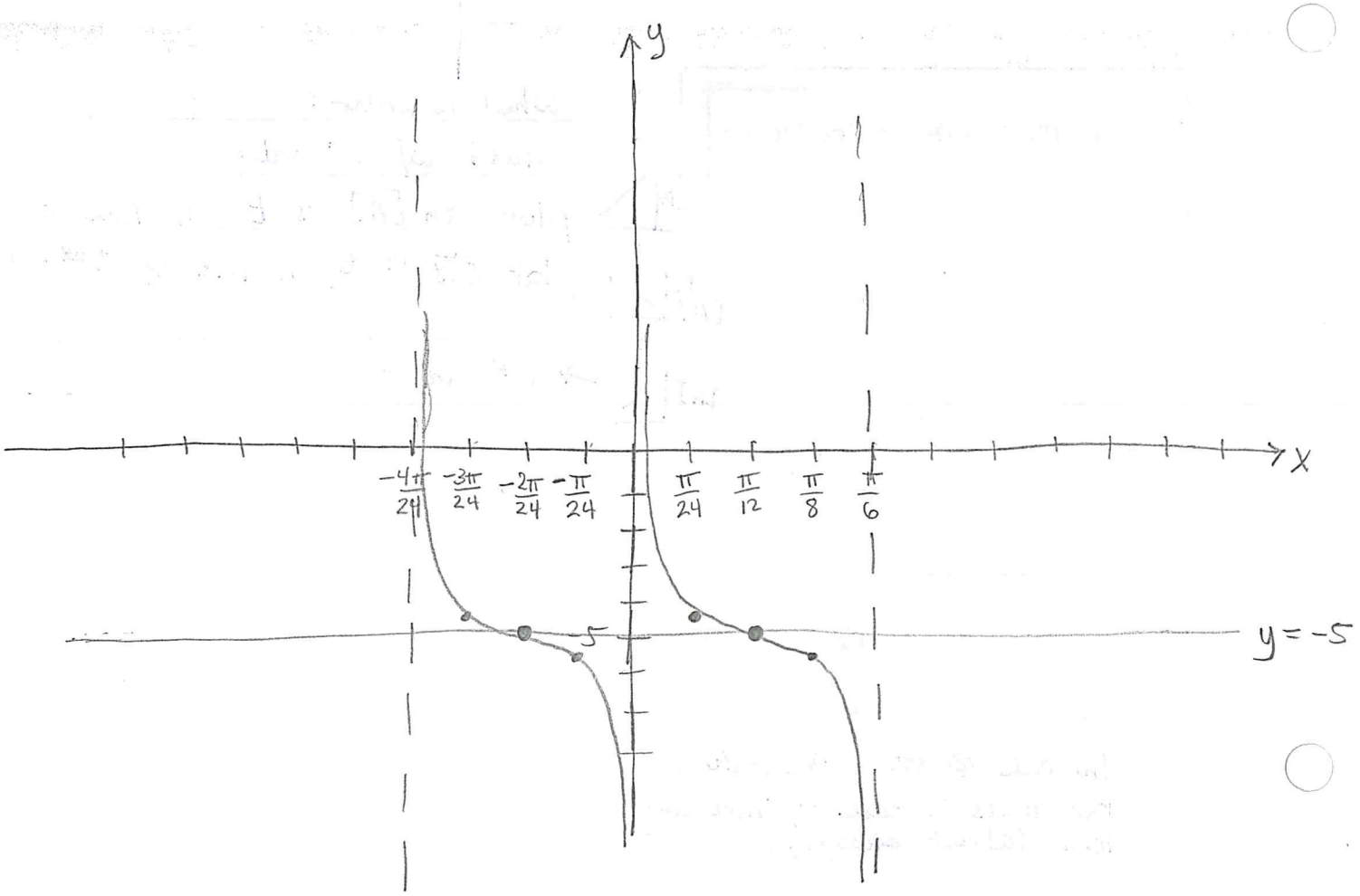
graph $y = 4\sin\left(\frac{\pi}{2}(\theta - 1)\right) + 2$

phase shift = +1

period = $\frac{2\pi}{\pi/2} = 2\pi \cdot \frac{2}{\pi} = 4$



(H) $y = \frac{1}{2}\cot(6\theta + \pi) - 5$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$



L.A. $6\theta + \pi = 0$

$6\theta = -\pi$

$\theta = -\frac{\pi}{6}$

$-\frac{4\pi}{24}$

R.A. $6\theta + \pi = \pi$

$6\theta = 0$

$\theta = 0$