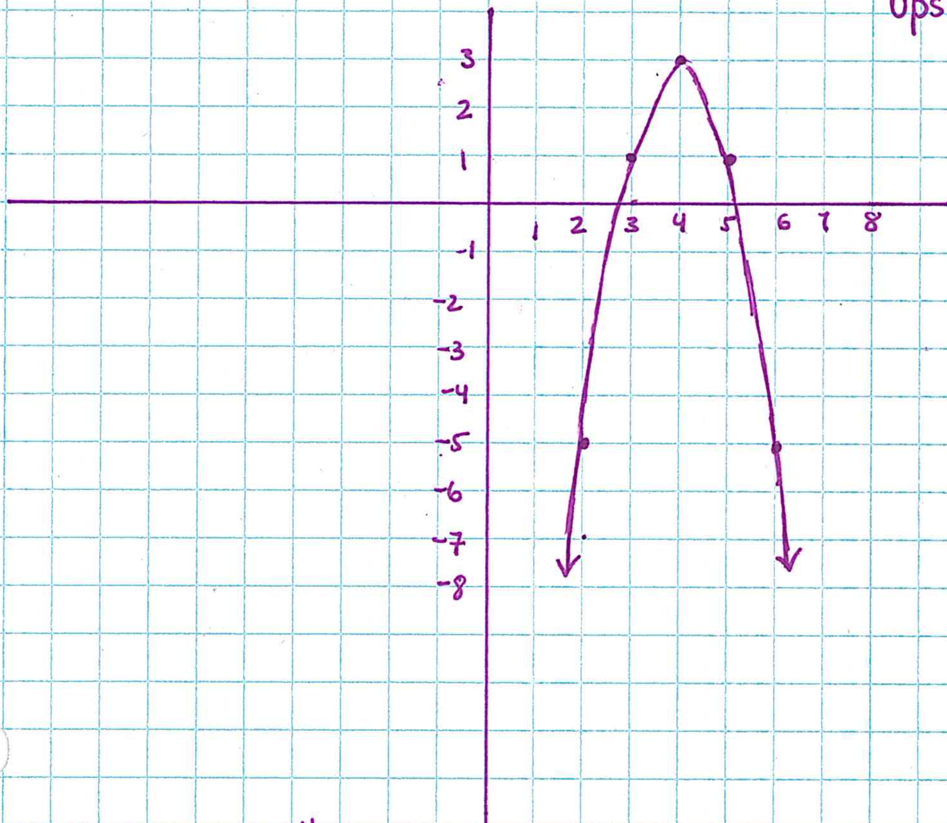
KEY

① $y = 3 - 2(4-x)^2$

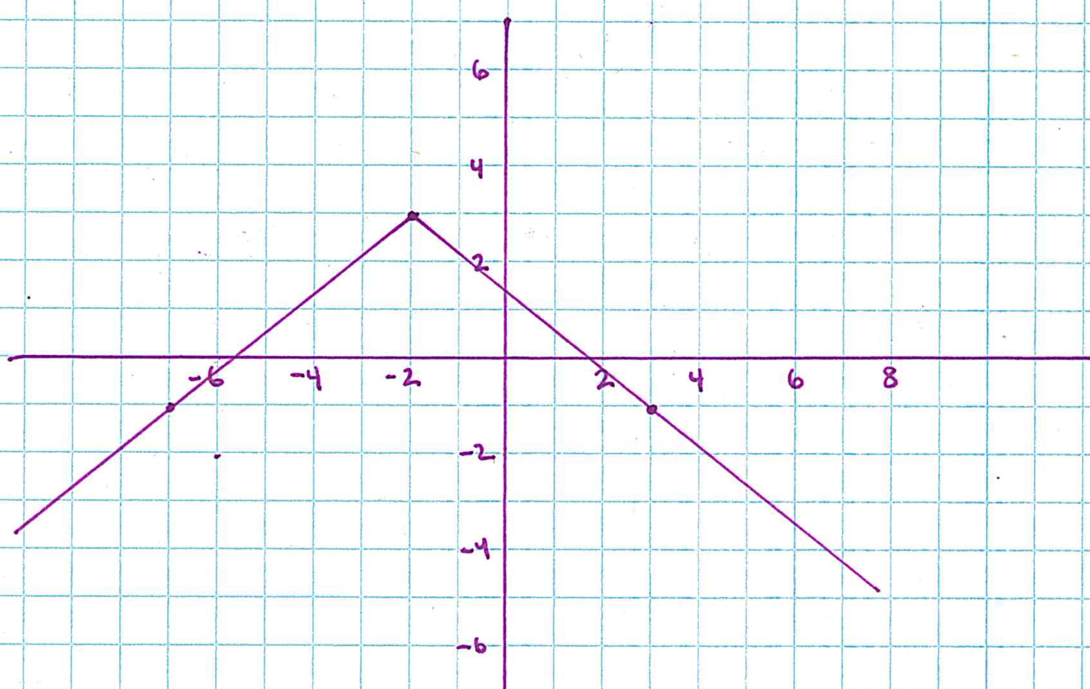
$y = -2(-(x-4))^2 + 3$

vertex at $(4,3)$
upside down

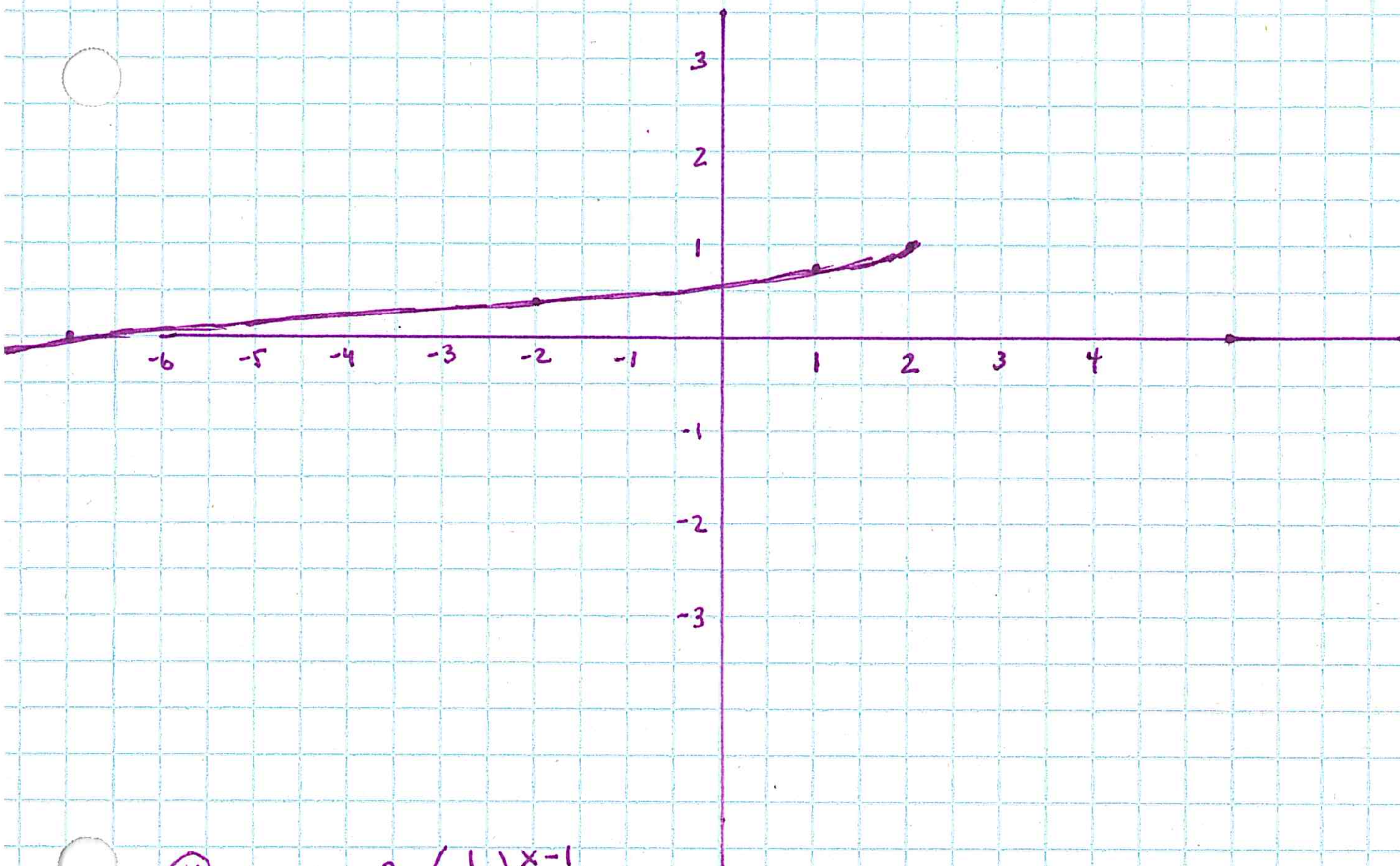


#40 was wrong in the original KEY.

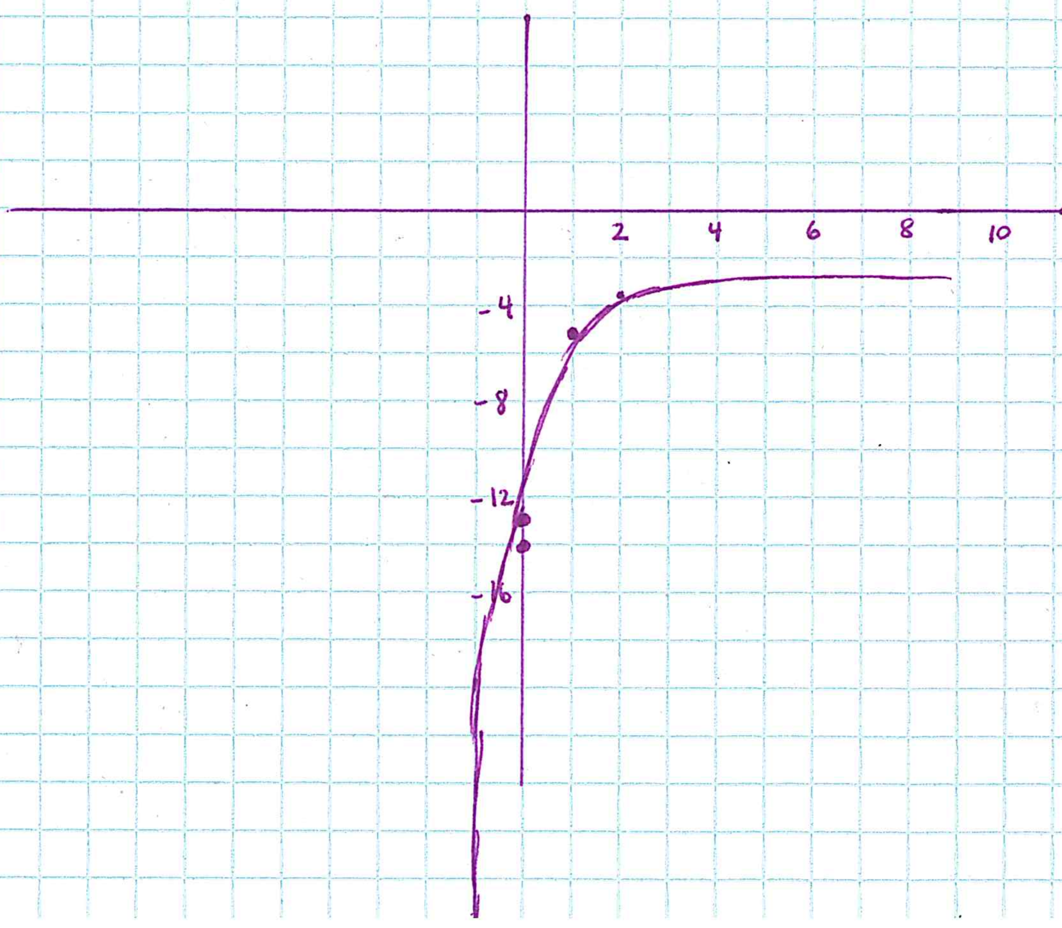
② $f(x) = -\frac{4}{5}|x+2| + 3$



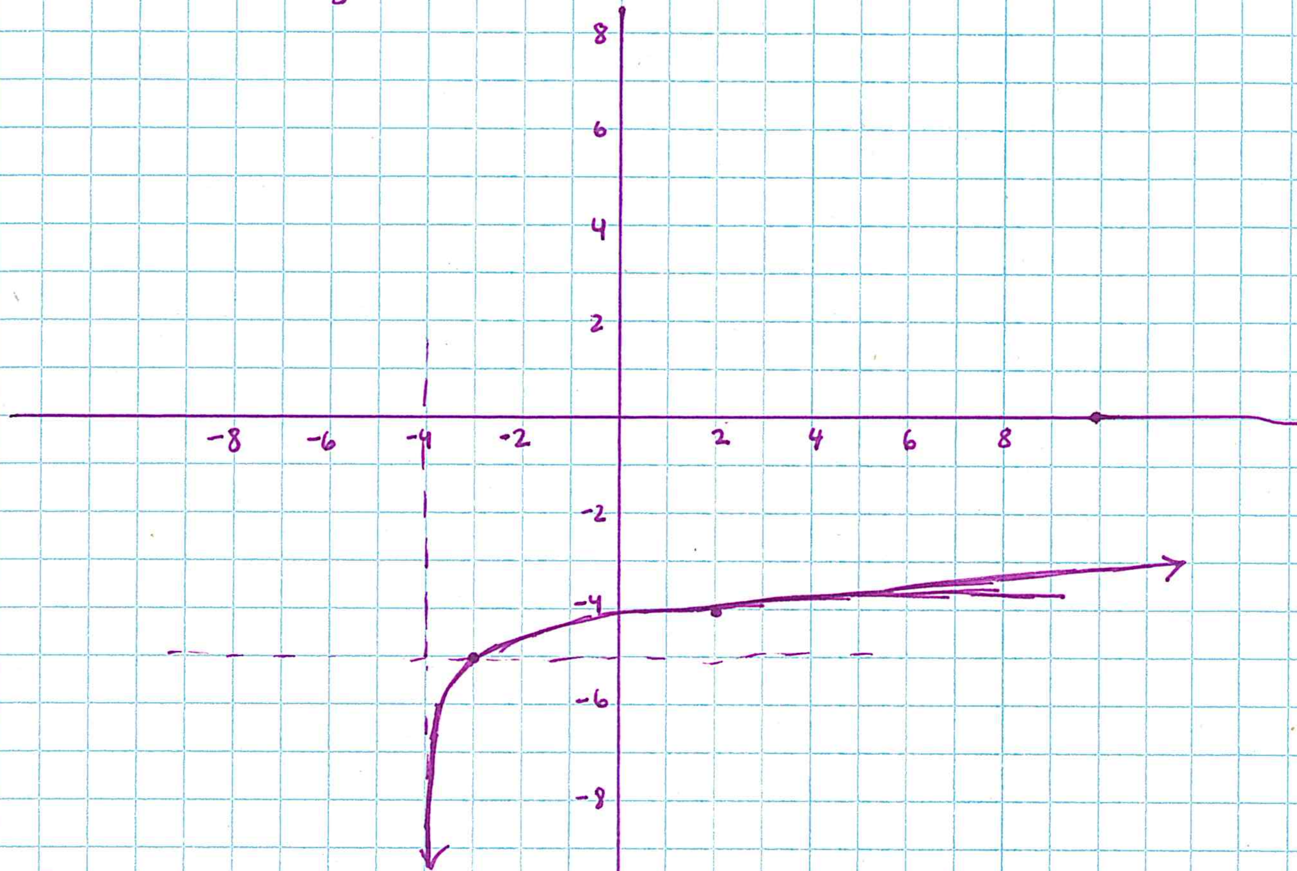
③ $y = -\frac{1}{3}\sqrt{2-x} + 1$ rewrite $y = -\frac{1}{3}\sqrt{-(x-2)} + 1$



④ $y = -2\left(\frac{1}{5}\right)^{x-1} - 3$



⑤ $f(x) = \log_6(x+4) - 5$



⑥ $y = \frac{1}{4}(x-1)^3 + 2$

start at $(1,2)$

pattern from start

over 1, up $\frac{1}{4}(1)$

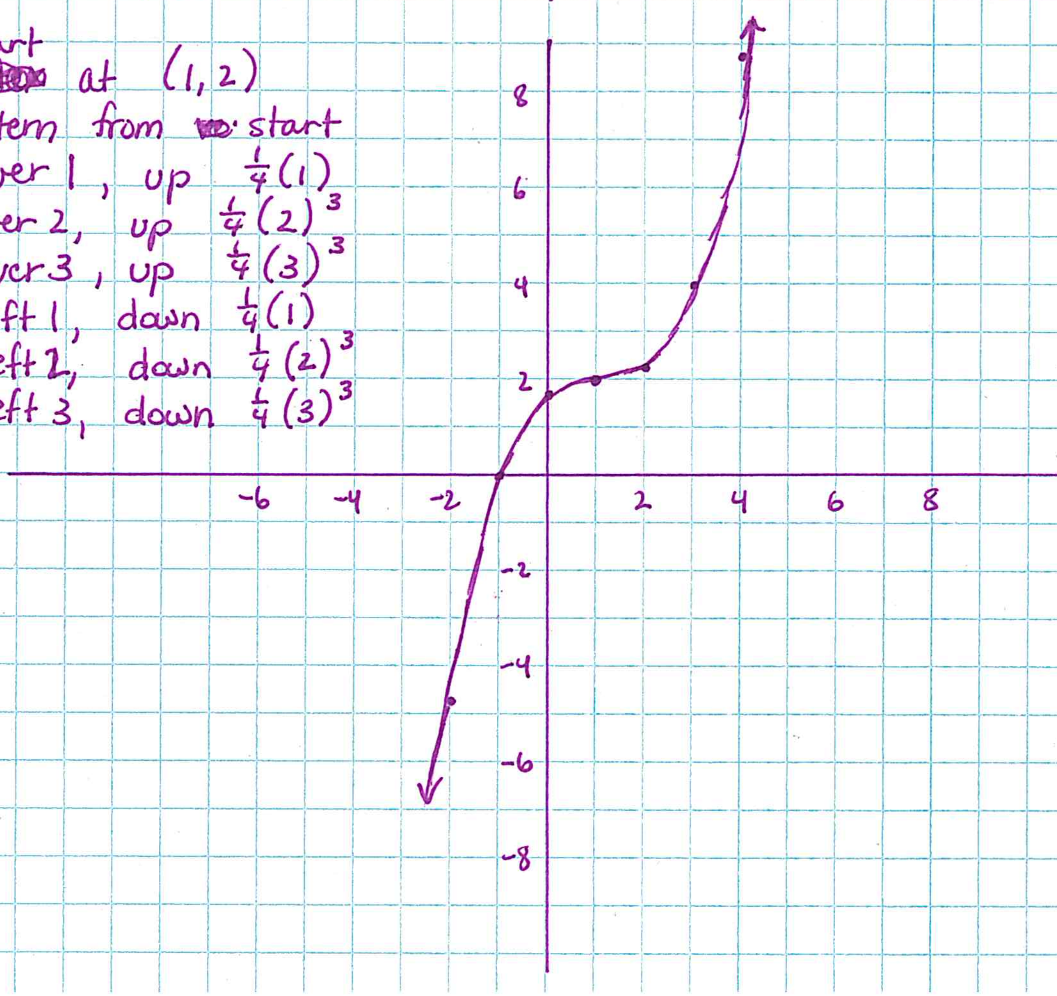
over 2, up $\frac{1}{4}(2)^3$

over 3, up $\frac{1}{4}(3)^3$

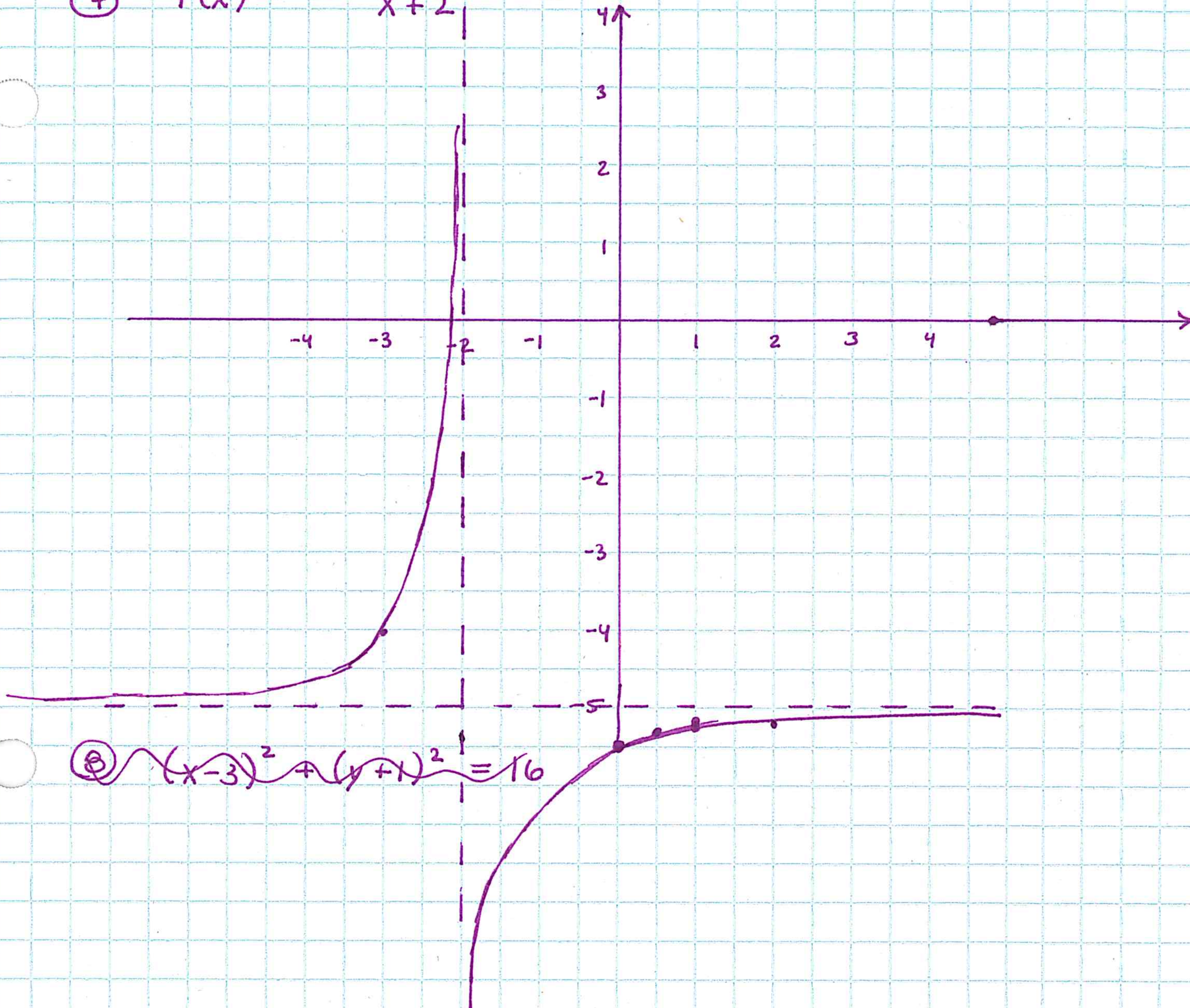
left 1, down $\frac{1}{4}(1)$

left 2, down $\frac{1}{4}(2)^3$

left 3, down $\frac{1}{4}(3)^3$



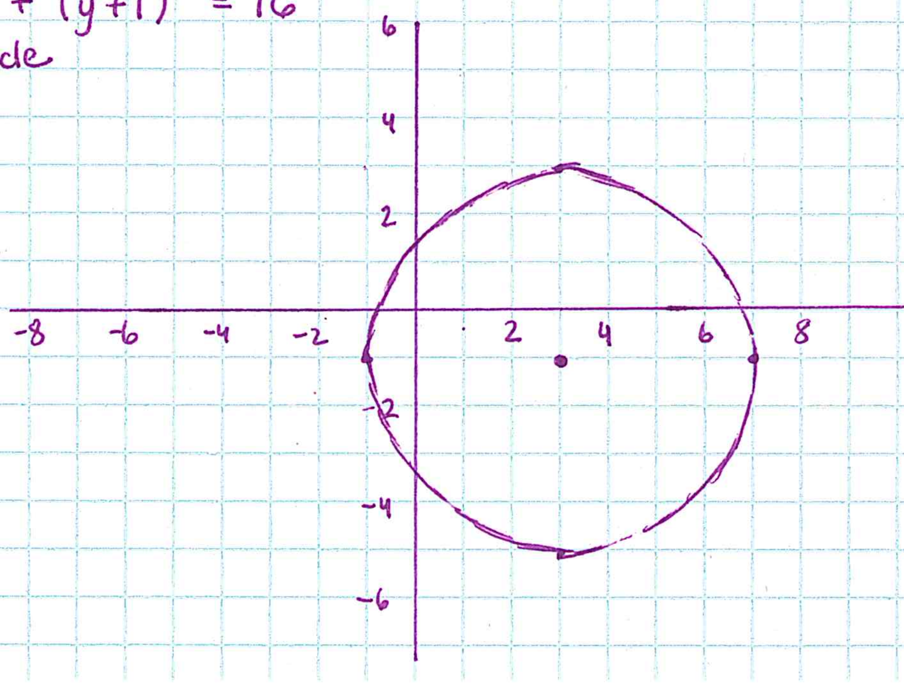
⑦ $f(x) = -\frac{1}{x+2} - 5$



~~⑧ $(x-3)^2 + (y+1)^2 = 16$~~

⑧ $(x-3)^2 + (y+1)^2 = 16$

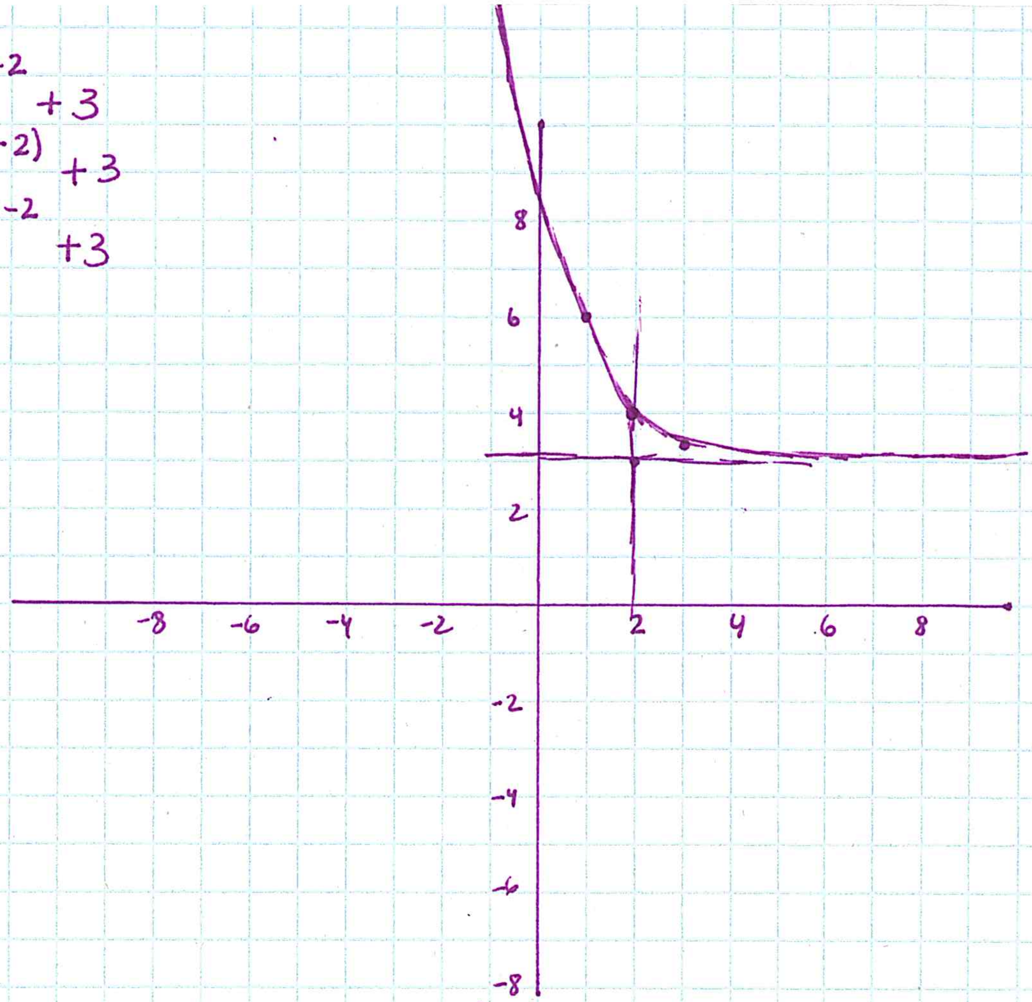
circle



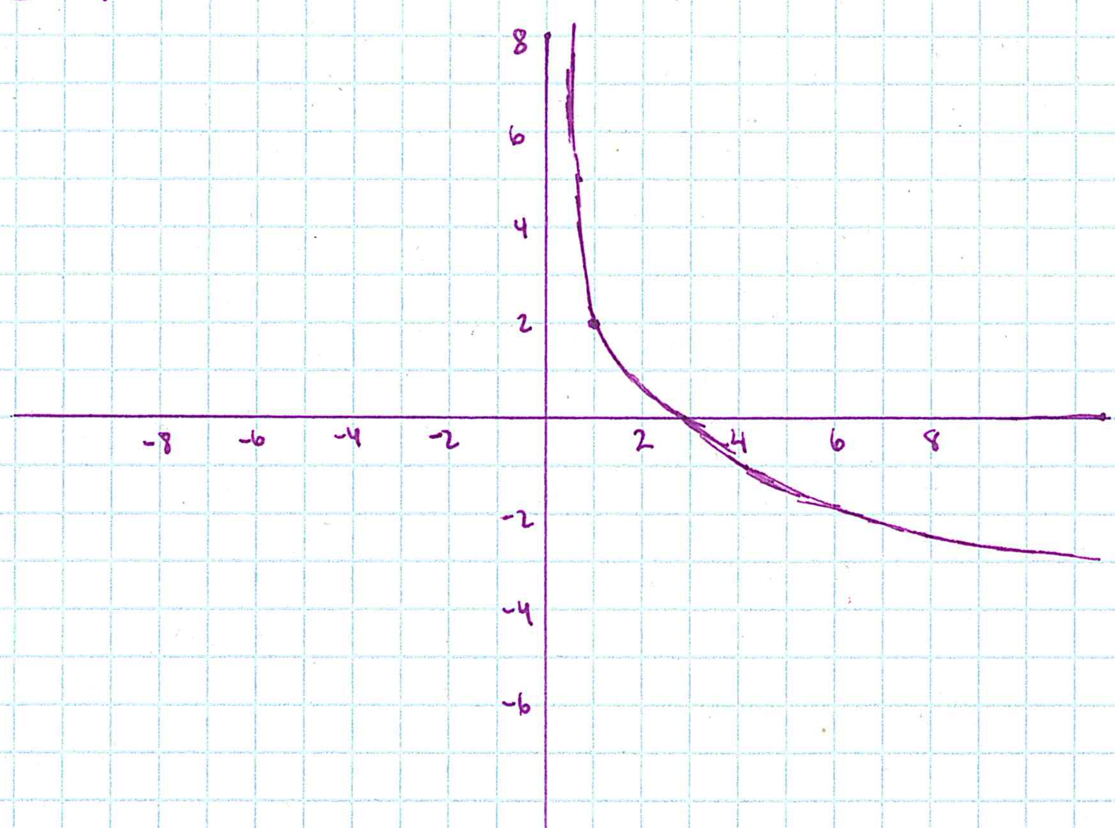
⑨ $f(x) = 3^{-x+2} + 3$

$f(x) = 3^{-(x-2)} + 3$

$= \left(\frac{1}{3}\right)^{x-2} + 3$



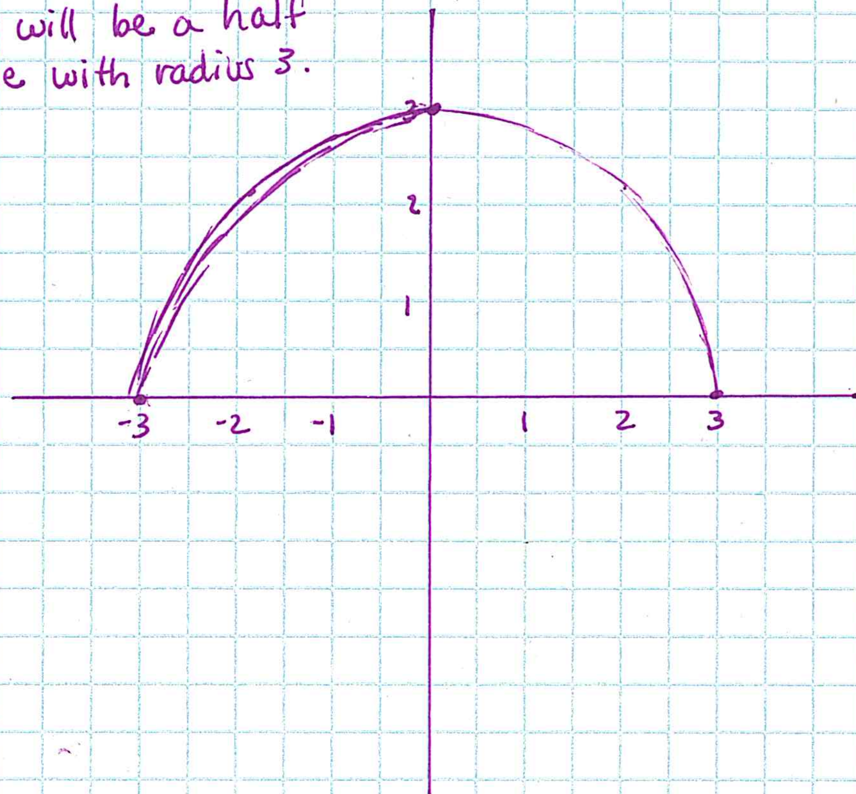
⑩ $y = -3\ln(x-1) + 2$



(11) $y = \sqrt{9-x^2}$

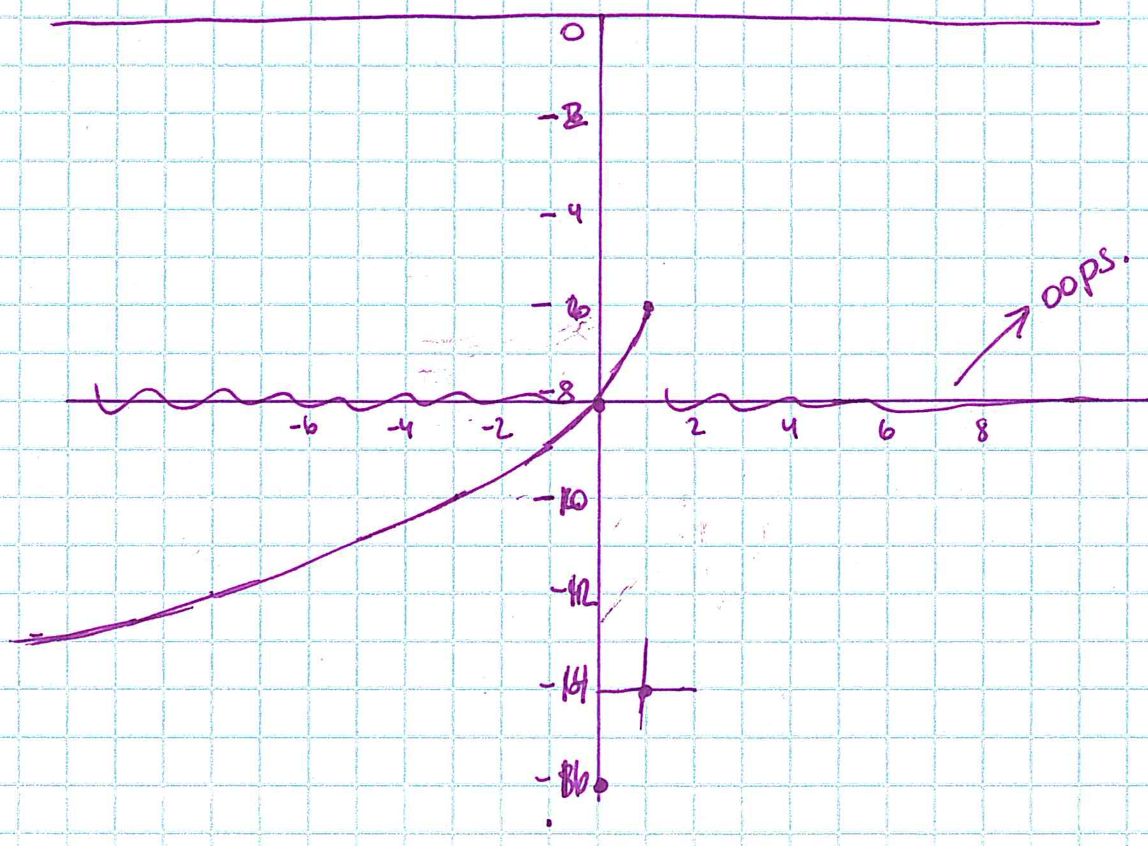
this will be a half circle with radius 3.

x	y
-3	0
-2	$\sqrt{5}$
0	3
2	$\sqrt{5}$
3	0



(12) $f(x) = -2\sqrt{-x+1} - 6$

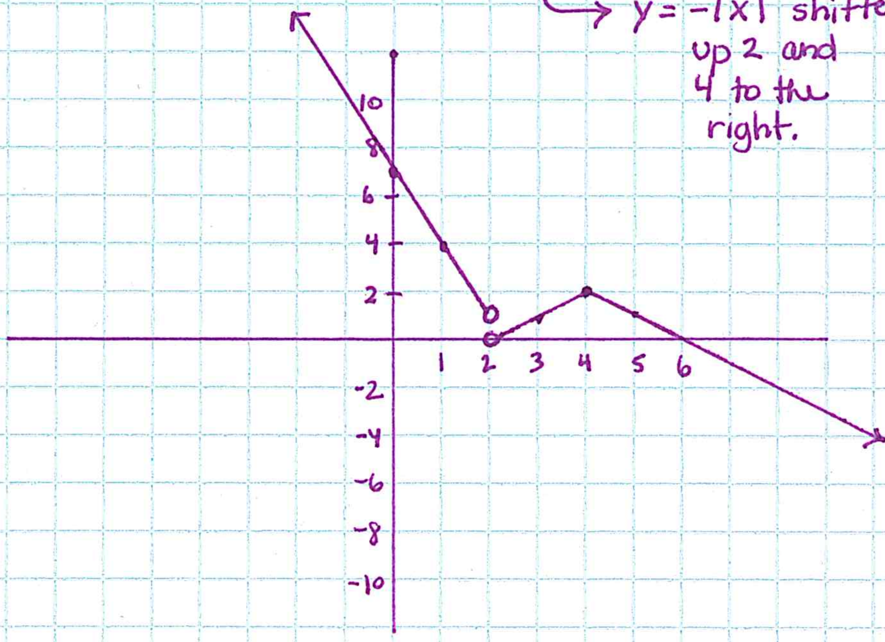
$= -2\sqrt{-(x-1)} - 6$



oops. pencil is better!

(13) $f(x) = \begin{cases} -3x+7 & x<2 \\ -|x-4|+2 & x>2 \end{cases}$

$\to$ $y=-|x|$ shifted up 2 and 4 to the right.



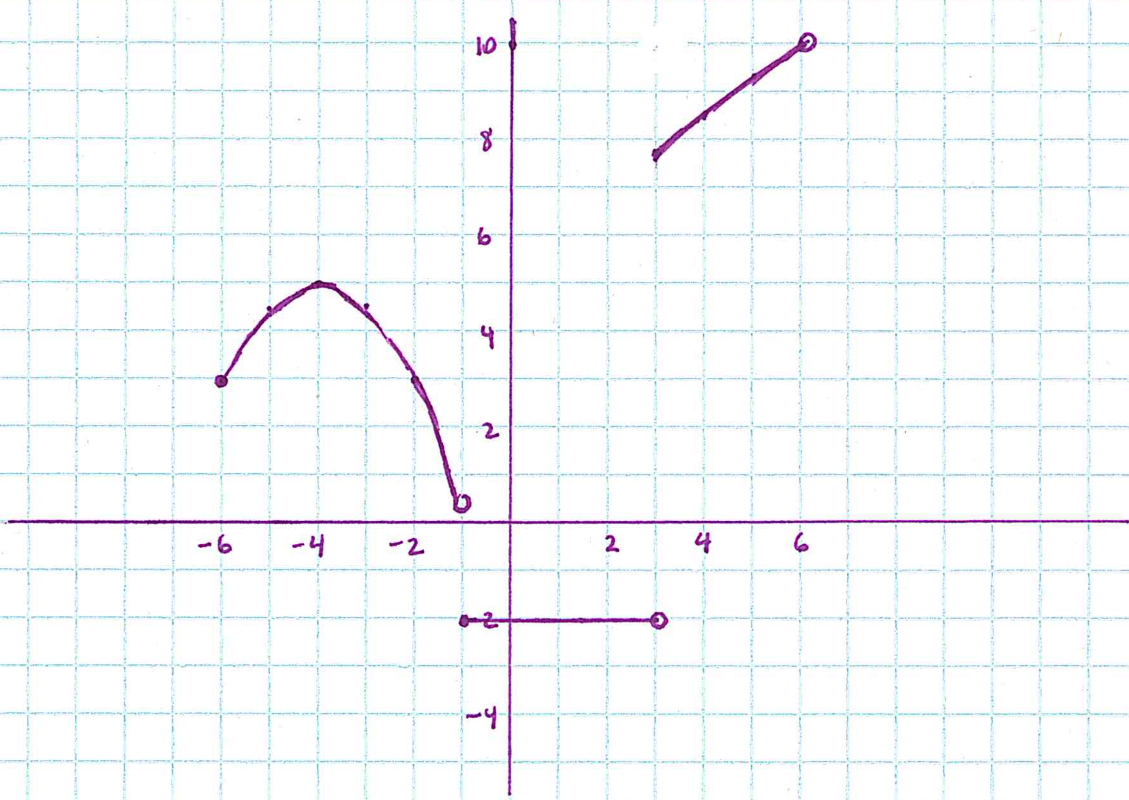
x	y
0	7
1	4

} use $y=-3x+7$ for these because $x<$

Domain:
$x \in (-\infty, 2) \cup (2, \infty)$

Range:
$y \in (-\infty, \infty)$

(14) $g(x) = \begin{cases} -\frac{1}{2}(x+4)^2+5 & -6 \le x < -1 \\ -2 & -1 \le x < 3 \\ 4\sqrt{x+3} - 2 & 3 \le x < 6 \end{cases}$

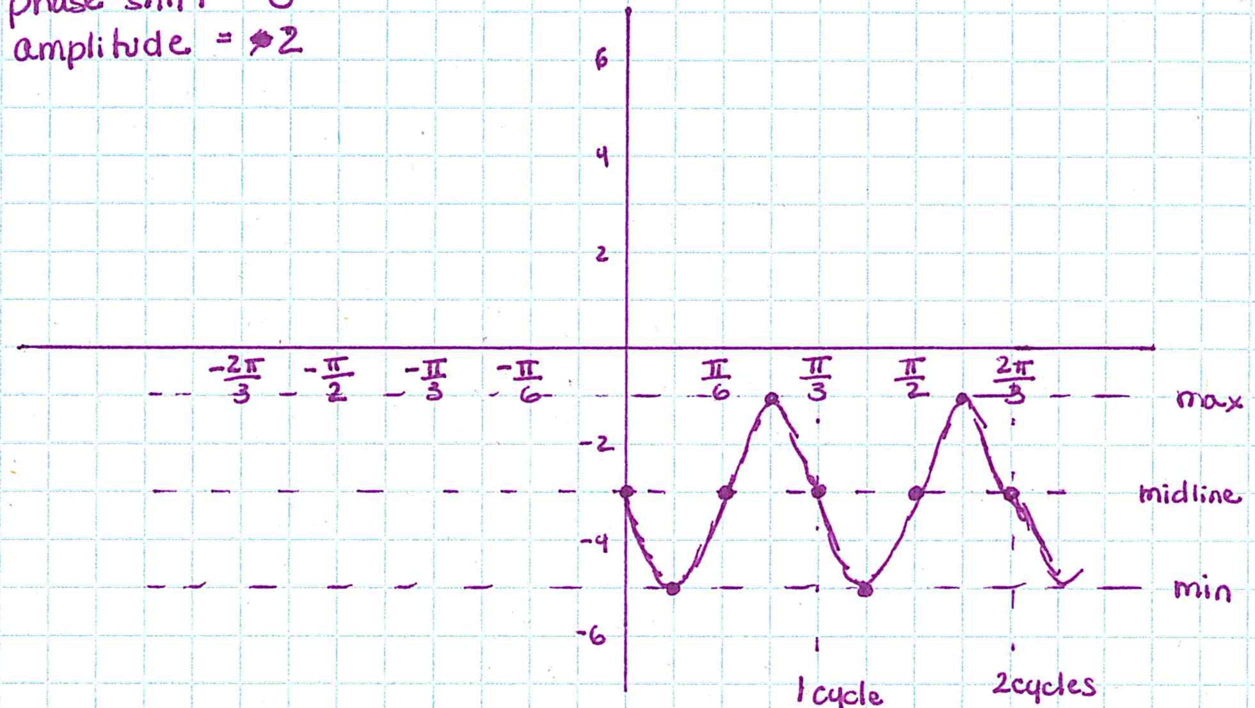


(15) $y = -2\sin 6x - 3$

period $= \frac{2\pi}{6} = \frac{\pi}{3}$

phase shift $= 0$

amplitude $= 2$

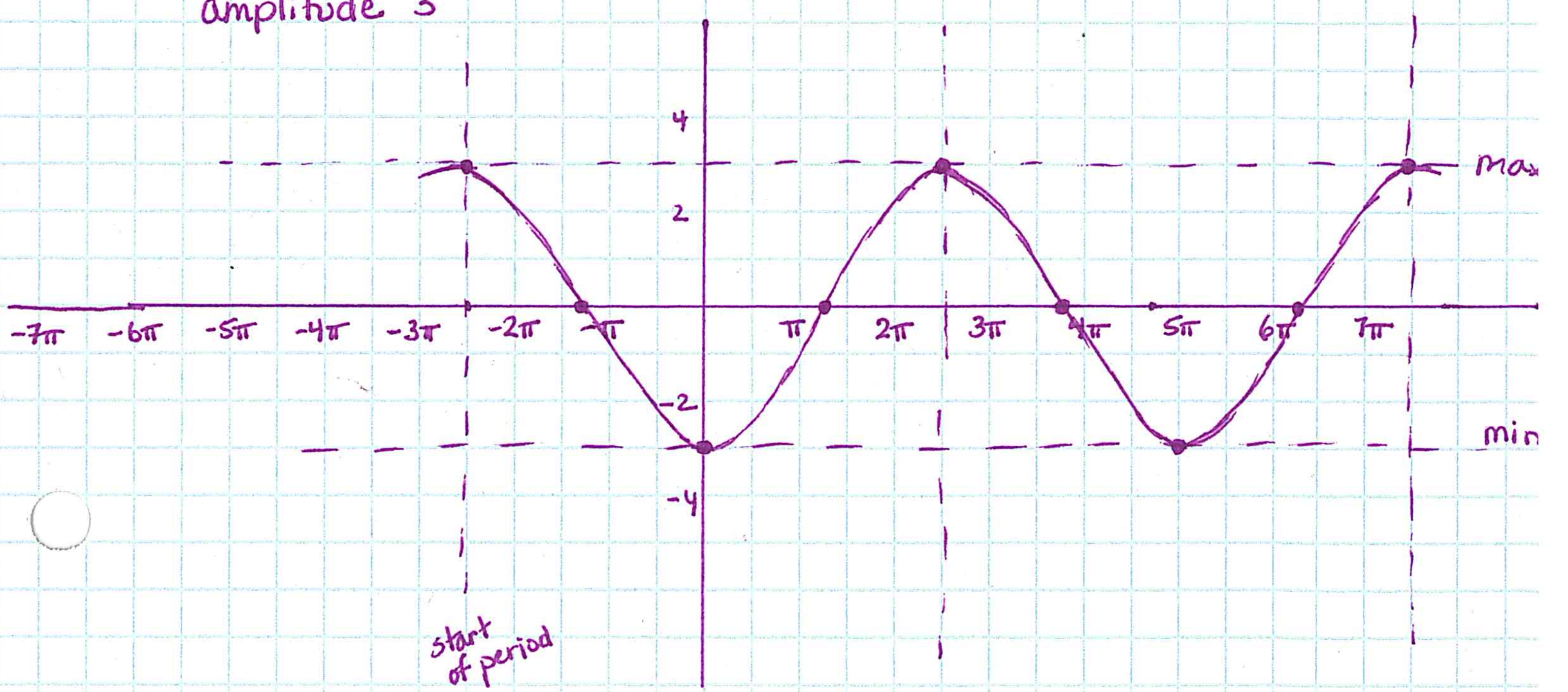


(16) $y = 3\cos\left(\frac{2x}{5} + \pi\right)$

period $= \frac{2\pi}{\frac{2}{5}} = 2\pi\left(\frac{5}{2}\right) = 5\pi$

phase shift $= -\frac{\pi}{\frac{2}{5}} = -\frac{5\pi}{2}$

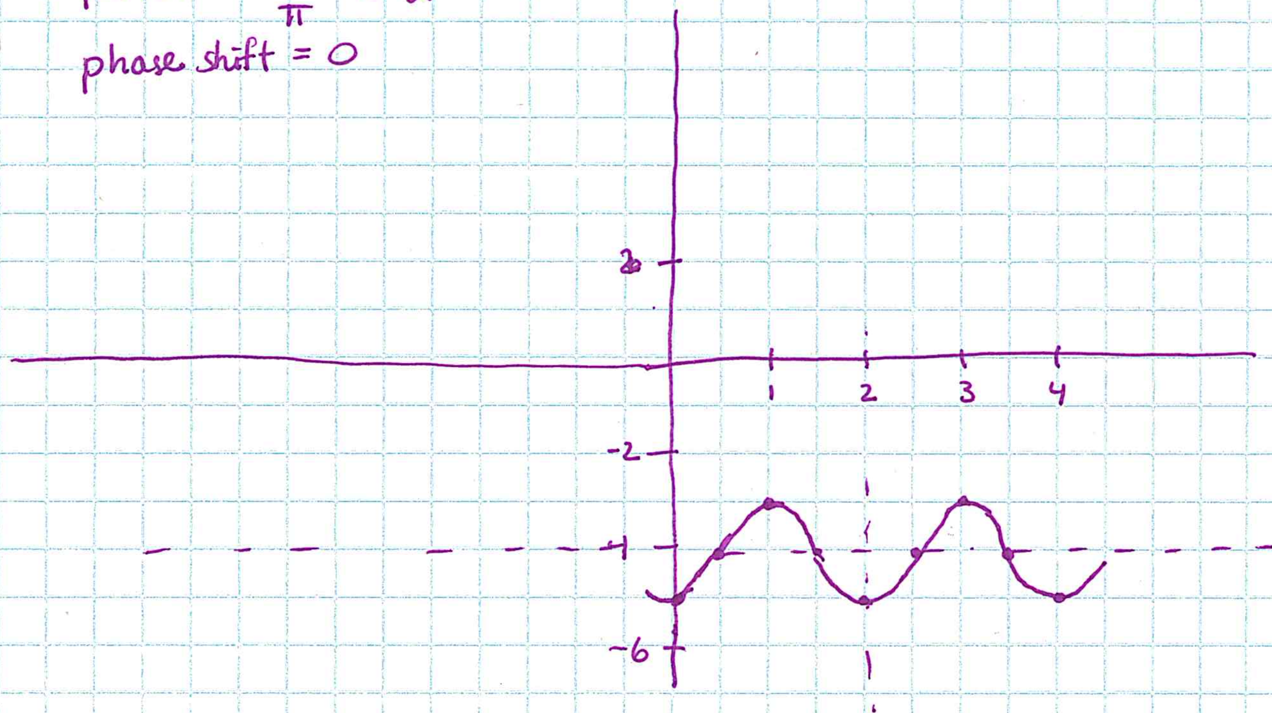
amplitude 3



(17) $y = -4 - \cos \pi x$

period $= \frac{2\pi}{\pi} = 2$

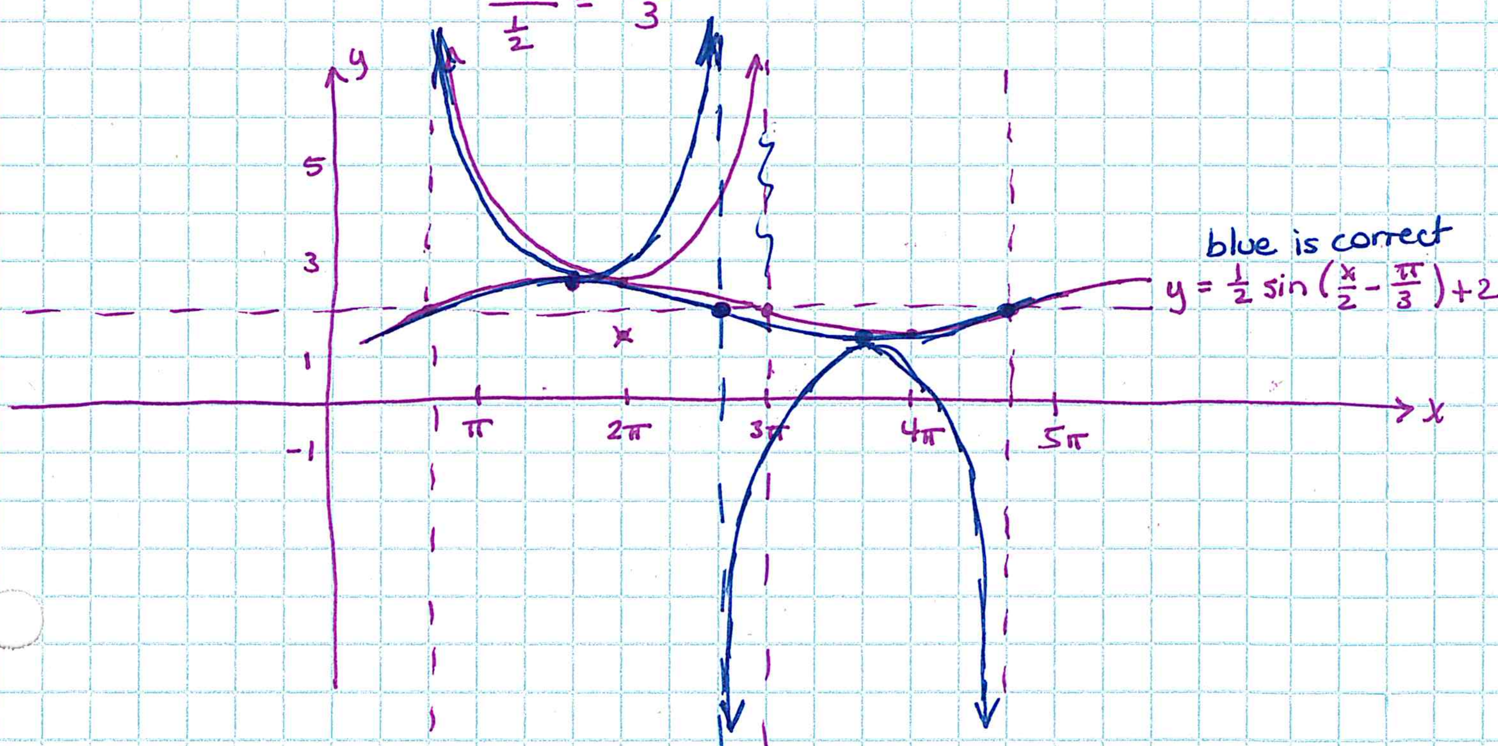
phase shift $= 0$



(18) $y = \frac{1}{2} \csc\left(\frac{x}{2} - \frac{\pi}{3}\right) + 2$

period $= \frac{2\pi}{1/2} = 4\pi$

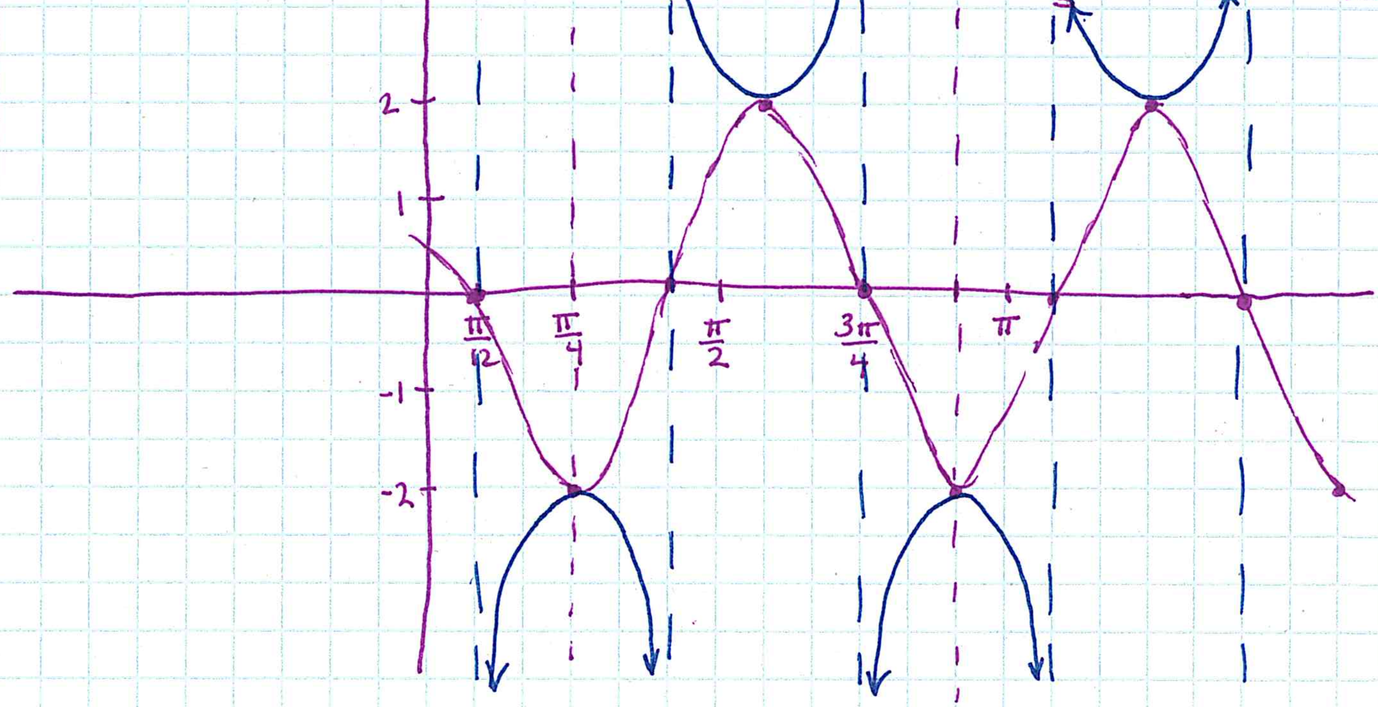
phase shift $= \frac{\frac{\pi}{3}}{\frac{1}{2}} = \frac{2\pi}{3}$



(19) $y = -2\sec\left(3x - \frac{3\pi}{4}\right)$

period $= \frac{2\pi}{3} = \frac{8\pi}{12}$

phase shift $= \dfrac{\frac{3\pi}{4}}{3} = \frac{\pi}{4} = \frac{3\pi}{12}$



(20) $y = \frac{2}{3}\tan\left(x + \frac{\pi}{4}\right) - 1$

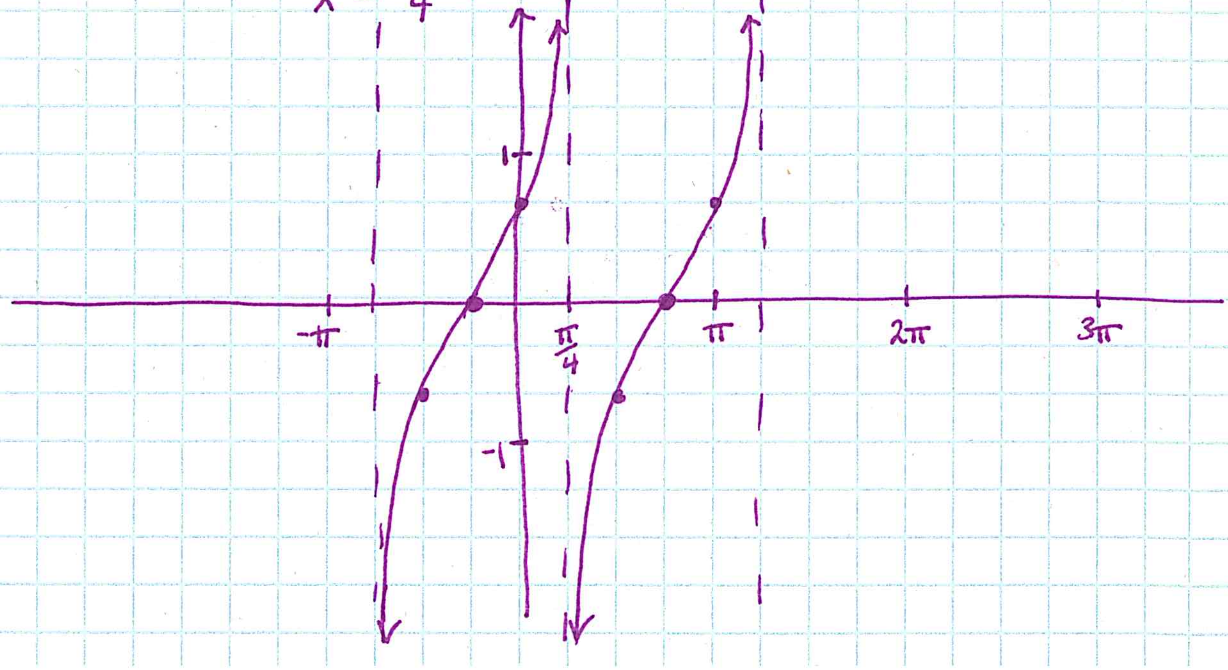
asymptote $\cancel{x} + \frac{\pi}{4} = -\frac{\pi}{2}$

$x = -\frac{3\pi}{4}$

$x + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} - \frac{\pi}{4}$

$x = \frac{\pi}{4}$



(21) $y = \cot\left(\frac{\pi x}{2} + \pi\right) + 3$

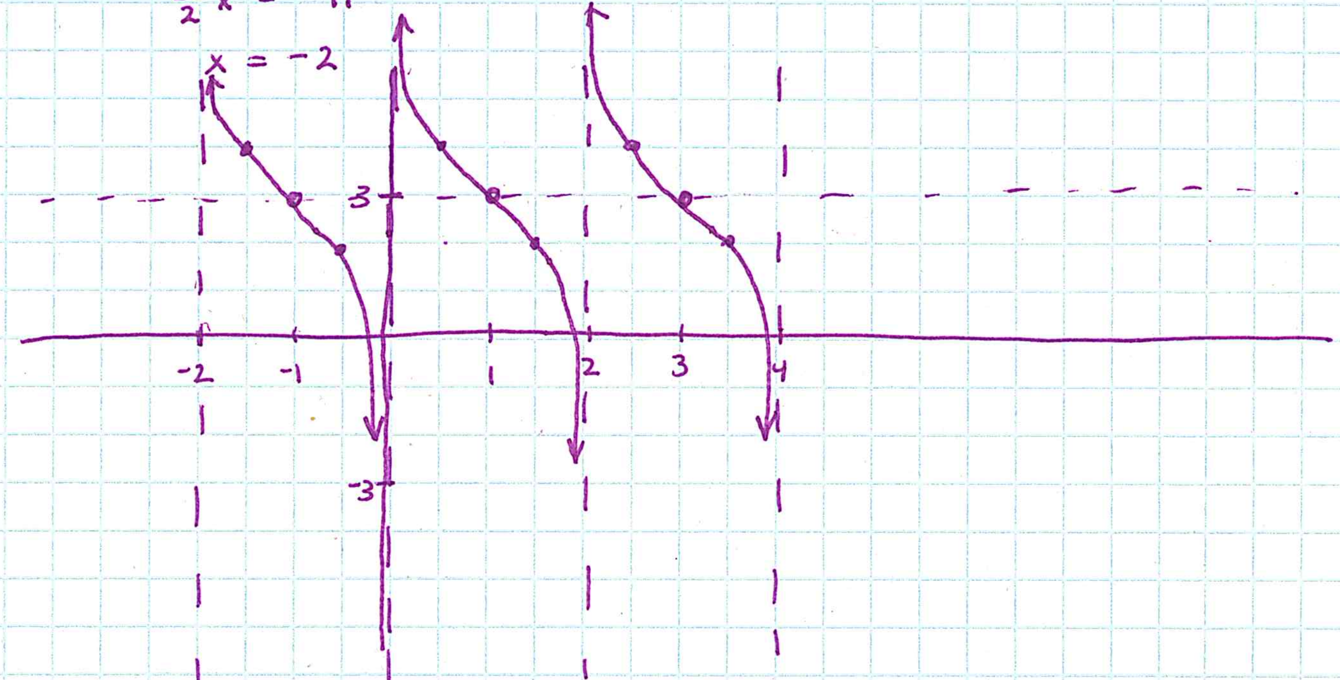
$\frac{\pi}{2}x + \pi = 0$

$\frac{\pi}{2}x = -\pi$

$x = -2$

$\frac{\pi}{2}x + \pi = \pi$

$x = 0$



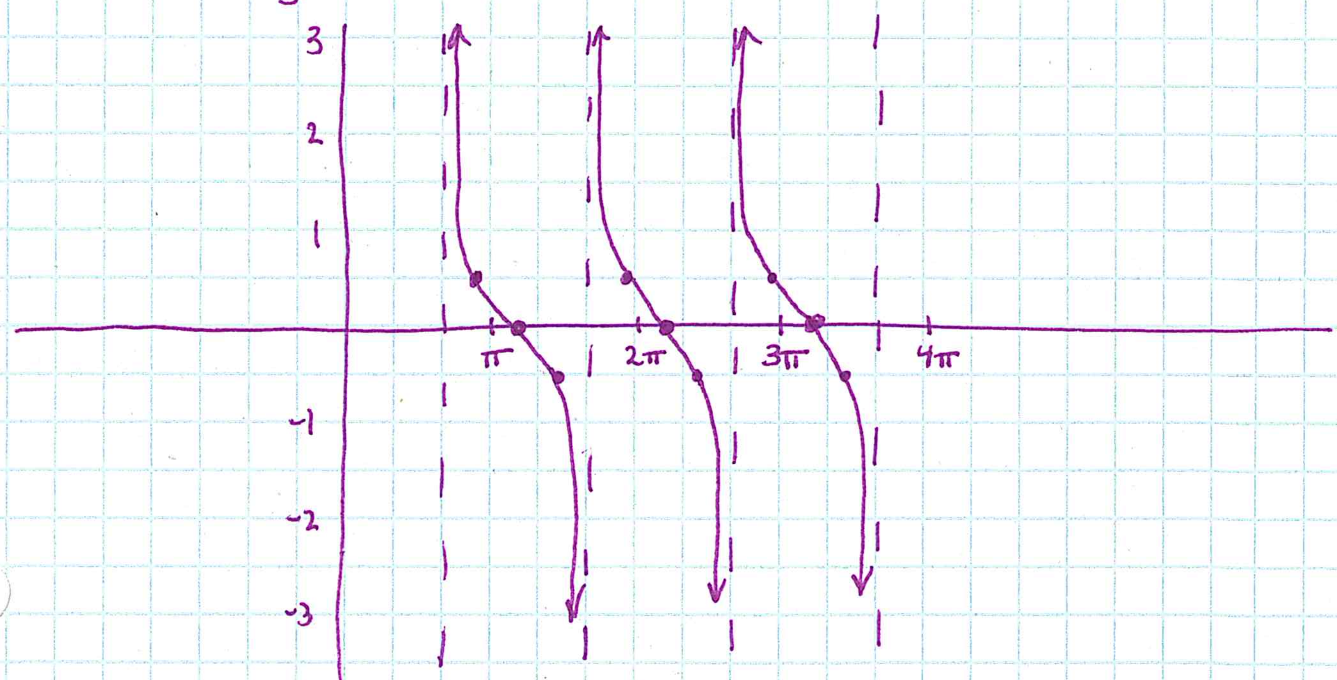
(22) $y = \frac{1}{2}\cot\left(x - \frac{2\pi}{3}\right)$

$x - \frac{2\pi}{3} = 0$

$x = \frac{2\pi}{3}$

$x - \frac{2\pi}{3} = \pi$

$x = \frac{5\pi}{3}$



(23) Find the domain & range

#2 $f(x) = -\frac{4}{5}|x+2| + 3$

domain : all real numbers $(-\infty, \infty)$

as $x \to -\infty$, $y \to -\infty$
as $x \to \infty$ $y \to -\infty$

range: notice that $-\frac{4}{5}|x+2|$ is always negative.
maximum y value will occur when $-\frac{4}{5}|x+2| = 0$
$y = 3$

$(-\infty, 3]$

#3. $y = -\frac{1}{3}\sqrt{2-x} + 1$

domain: $2-x \geq 0$ $(-\infty, 2]$
$x \leq 2$

range: notice $-\frac{1}{3}\sqrt{2-x}$ is always negative & gets ~~larger~~ smaller for ~~larger~~ smaller values of x
max y value will be 1 (when $\sqrt{2-x} = 0$)

$(-\infty, 1]$

as $x \to -\infty$
$y \to -\infty$

#7. $f(x) = -\frac{1}{x+2} - 5$

domain $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

range as $x \to -\infty$ $y \to -5$
$x \to +\infty$ $y \to -5$ $(-\infty, -5) \cup (-5, \infty)$
as $x \to -2.0001$ $y \to \infty$
as $x \to -1.9999$ $y \to -\infty$

#11. $y = \sqrt{9-x^2}$

domain : $9 - x^2 \geq 0$
$x \leq \pm 3$ $[-3, 3]$
$-3 \leq x \leq 3$

range: $\sqrt{9-x^2}$ is always positive ≥ 0
at $x = \pm 3$ $y = 0$ at $x = 0$ $y = 3$
$[0, 3]$

(24) Domain $(-\infty, \infty)$

Range $[-3, \infty)$ by reading graph

(25) Domain $(-\infty, 4) \cup (4, \infty)$

Range $[-2, \infty)$

(26) $f(x) = x^2 - 2x - 3$

If a function is odd $f(-x) = -f(x)$
If a function is even $f(-x) = f(x)$

$$f(-x) = (-x)^2 - 2(-x) - 3$$

$$= +x^2 + 2x - 3$$ Neither even nor odd

$$\neq f(x)$$
$$\neq -f(x)$$

(27) $f(x) = \dfrac{x^3}{8+x^2}$

$$f(-x) = \frac{(-x)^3}{8+(-x)^2}$$

$$= \frac{-x^3}{8+x^2}$$

$$= -\left[\frac{x^3}{8+x^2}\right] = -f(x)$$ odd function

(28) $f(x) = e^{x^2}$

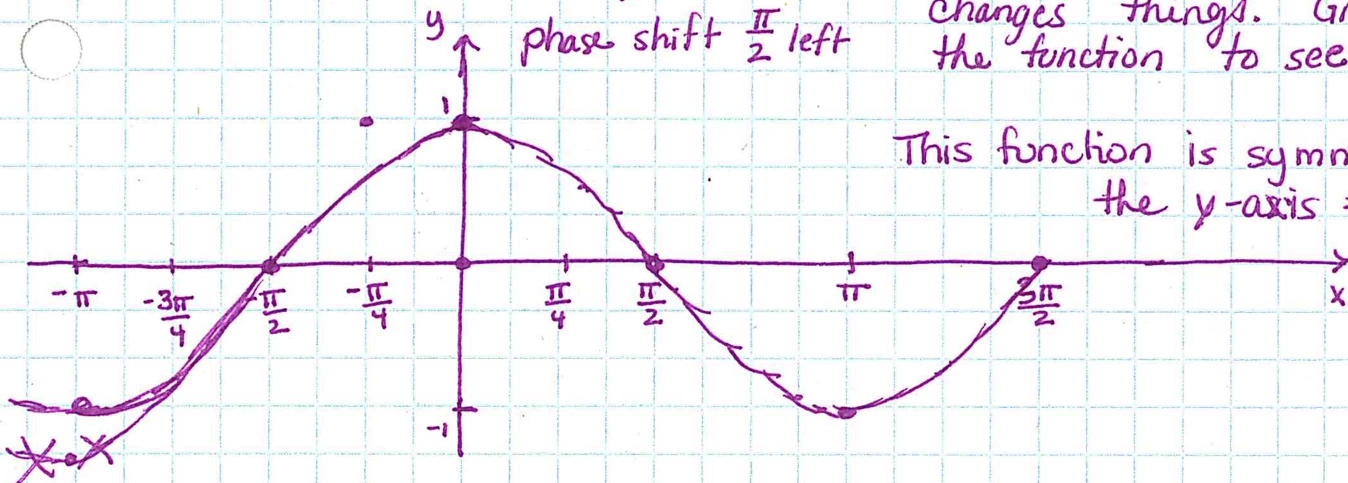
$$f(-x) = e^{(-x)^2}$$

$$= e^{x^2} = f(x)$$ even function

(29) $f(x) = \sin\left(x + \frac{\pi}{2}\right)$

phase shift $\frac{\pi}{2}$ left

sin(x) is an odd function, but the $\frac{\pi}{2}$ phase shift changes things. Graph the function to see symmetry



This function is symmetric about the y-axis ⇒ <u>EVEN</u>

(30) $y = \dfrac{x+3}{2x-7}$

change x and y
solve for y

$$x = \frac{y+3}{2y-7}$$

$$x(2y-7) = y+3$$

$$2xy - 7x = y + 3$$

$$\cancel{-y = 3 + 7x - 2xy}$$

$$2xy - y = 7x + 3$$

$$(2x-1)y = 7x+3$$

$$\boxed{f^{-1}(x) = \frac{7x+3}{2x-1}}$$

this is a function

(31) $y = 5 + 2\log_3(x+5)$

$x = 5 + 2\log_3(y+5)$

$\frac{x-5}{2} = \log_3(y+5)$

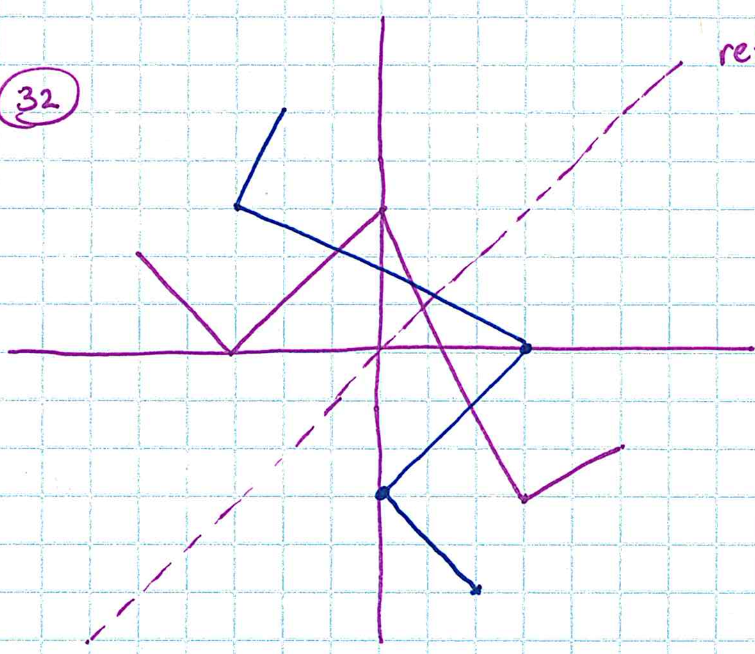
$3^{\left(\frac{x-5}{2}\right)} = y+5$

$\boxed{f^{-1}(x) = 3^{\frac{x-5}{2}} - 5}$

this is an exponential function

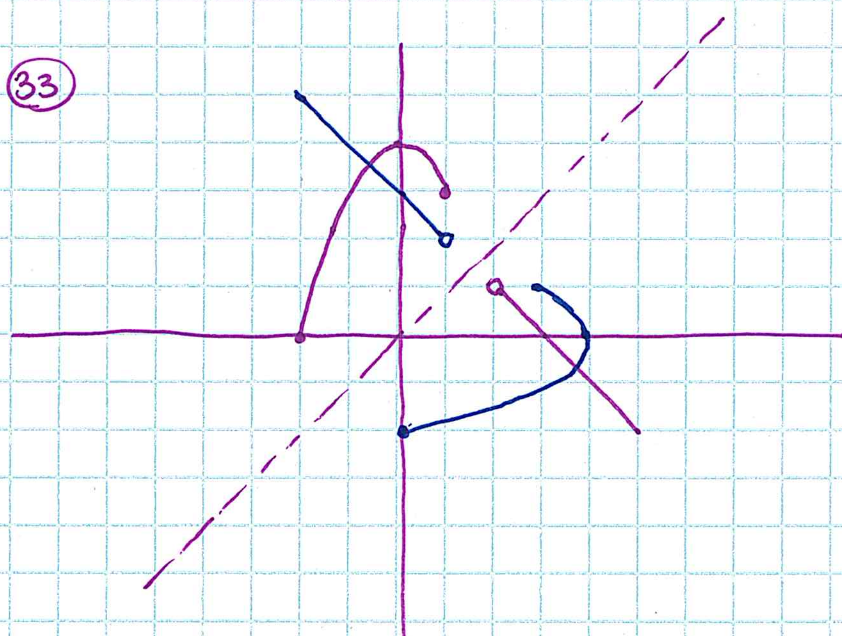
reflect across $y=x$

(32)



blue is $f^{-1}(x)$

(33)



blue is $f^{-1}(x)$

$$f(x) = \frac{2+5x}{5x-2}$$

$$g(x) = \sqrt{9-x^2}$$

$$h(x) = x^2 - 9$$

(34) $(h \circ g)(x) = h\left(\sqrt{9-x^2}\right)$

$$= \left(\sqrt{9-x^2}\right)^2 - 9$$

$$= -x^2 \qquad -3 \le x \le 3$$

(35) $f(g(x)) = f\left(\sqrt{9-x^2}\right)$

$$= \frac{2+5\left(\sqrt{9-x^2}\right)}{5\left(\sqrt{9-x^2}\right)-2}$$

(36) $f(g(h(x))) = f(g(x^2-9))$

$$= f\left(\sqrt{9-(x^2-9)^2}\right)$$

$$= \cancel{f(} \frac{2+5\left(\sqrt{9-(x^2-9)^2}\right)}{5\left(\sqrt{9-(x^2-9)^2}\right) - 2}$$

(37) $(f \circ f)(x) = f\left(\frac{2+5x}{5x-2}\right)$

$$= \frac{2+5\left(\frac{2+5x}{5x-2}\right)}{5\left(\frac{2+5x}{5x-2}\right)-2} = \frac{\frac{10x-4+10+25x}{\cancel{5x-2}}}{\frac{\cancel{25x}\,10+25x-10x+4}{\cancel{5x-2}}}$$

$$\boxed{= \frac{35x+6}{15x+14}}$$

(38) (a) $f(f^{-1}(x)) = f\left(\frac{7x+3}{2x-1}\right)$

$$= \frac{\frac{7x+3}{2x-1}+3}{2\left(\frac{7x+3}{2x-1}\right)-7}$$

$$= \frac{\frac{7x+3+6x-3}{\cancel{2x-1}}}{\frac{14x+6-14x+7}{\cancel{2x-1}}}$$

$$= \frac{13x}{13} = \underline{\underline{x}}$$

Try it the other way too $f^{-1}(f(x))$

(b) $f(f^{-1}(x)) = f\left(3^{\frac{x-5}{2}} - 5\right)$

$$= 5 + 2\log_3\left(3^{\frac{x-5}{2}} \cancel{-5} \cancel{+5}\right)$$

$$= 5 + 2\left(\frac{x-5}{2}\right)$$

$$= 5 + x - 5$$

$$\underline{\underline{= x}}$$

since $f(f^{-1}(x)) = x$ the two functions are inverses!

(39) $(5x^2z^6)^3(5x^2yz^{-2})^{-3} = \cancel{5^3}\cancel{x^6}z^{18}\cdot\cancel{5^{-3}}\cancel{x^{-6}}y^{-3}z^6$

$= \dfrac{z^{24}}{y^3}$

(40) $\dfrac{2(x^{-3}y^4)^{-2}}{(2x^{-1}y^5z)^2} = \dfrac{2x^{+6}y^{-8}}{2^2x^{-2}y^{10}z^2} = \cancel{\boxed{\dfrac{x^4}{2x^4y^{18}z^2}}}\ \dfrac{x^8}{2y^{18}z^2}$

$2^{1-2}x^{+6-(-2)}y^{-8-10}z^{-2}$

$2^{-1}x^8y^{-18}z^{-2}$

correct answer

(41) $3\ln x - \ln(x+3) + 2\ln y$

$\boxed{= \ln\left(\dfrac{x^3y^2}{x+3}\right)}$

(42) $\log\left(\dfrac{(x-1)^3}{y^2z}\right) = \log(x-1)^3 - \log(y^2z)$

$= 3\log(x-1) - \log y^2 - \log z$

$\boxed{= 3\log(x-1) - 2\log y - \log z}$

(43) $y = A\left(\frac{1}{2}\right)^{\frac{t}{21.77}}$ at $t=19$ $y = A\left(\frac{1}{2}\right)^{19/21.77}$

$y = .546A$

there will be 54.6% of Actinium left after 19 yrs.

(44) starting amount 250 $\quad$ t = time

x = ~~amount~~ growth factor

at 1 hr $\quad \dfrac{250x}{500x} = \dfrac{250 \cdot x^1}{250 \cdot x^{10}}$

at 10 hr

$$\frac{1}{2} = \frac{1}{x^9}$$

$$x^9 = 2$$

$$x = \sqrt[9]{2}$$

$$y = 250\left(\sqrt[9]{2}\right)^t$$

$$y = 250\left(\sqrt[9]{2}\right)^6$$

$y = \underline{396.9 \text{ bacteri}}$

(45) $y = 28.754(1.1195)^x$ from calculator (using $x = 1$ for 2011)

$$80 = 28.754(1.1195)^x$$

$x = 9.06$ years

spending will reach \$80B in 2020

(46)

$$52 = 24e^{3x-1} + 7$$
$$-7 \qquad\qquad -7$$

$$\frac{1}{24}(45) = (\cancel{24}e^{3x-1})\frac{1}{\cancel{24}}$$

$$\frac{45}{24} = e^{3x-1}$$

$$\ln\left(\frac{45}{24}\right) = 3x - 1$$

$$3x = \ln\left(\frac{45}{24}\right) + 1$$

$$x = \frac{\ln\left(\frac{45}{24}\right) + 1}{3}$$

$$\boxed{x = 0.543}$$

(47)

$$50 = 2(9)^{-5/x+4} - 4$$

$$54 = 2(9)^{-5/x+4}$$

$$27 = 9^{-5/x+4}$$

$$\log_9 27 = \frac{-5}{x+4}$$

$$1.5 = -\frac{5}{x+4}$$

$$1.5(x+4) = -5$$

$$x + 4 = -5\left(\frac{2}{3}\right)$$

$$x = -\frac{10}{3} - 4$$

$$\boxed{x = -\frac{22}{3}}$$

(48) $-6 = -3\log_9\left(\frac{x}{4}+2\right)$

$2 = \log_9\left(\frac{x}{4}+2\right)$

$9^2 = 9^{\log_9\left(\frac{x}{4}+2\right)}$

$81 = \frac{x}{4}+2$

$79 = \frac{x}{4}$

$\boxed{x = 316}$

(49) $-5 = \log_x 8 + 2$

$-7 = \log_x 8$

$x^{-7} = x^{(\log_x 8)}$

$x^{-7} = 8$

$\left(\frac{1}{x}\right)^7 = 8$

$\frac{1}{x} = \sqrt[7]{8}$

$\boxed{x = \frac{1}{\sqrt[7]{8}}}$

(50) $4\sin^2 x + 7 = 9 \quad 0 \le x \le 2\pi$

$4\sin^2 x = 2$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(51) $4\cos^2 x - 3 = 0 \quad -\infty < x < \infty$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{7\pi}{6},$

$\frac{5\pi}{6}, \frac{11\pi}{6}, \ldots$

$x = \cancel{n\pi \pm \frac{\pi}{6}}$

$x = n\pi - \frac{\pi}{6}, \; n\pi - \frac{5\pi}{6} \quad 0 \le x < 2$

(52) $(2\sin x - 1)(\cos x) = 0 \quad -\pi \le x \le \pi$

$\cancel{2\sin x\cos x - \cos x = 0}$

$2\sin x - 1 = 0$ or $\cos x = 0$

$2\sin x = 1 \qquad x = \frac{-\pi}{2}, \frac{\pi}{2}$

$\sin x = \frac{1}{2}$

or

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-\pi}{2}, \frac{\pi}{2}$

possible solutions

(53) $(\tan x)(\cos x - 1)(\sin x + 1) = 0$

$\tan x = 0$	$\cos x - 1 = 0$	$\sin x + 1 = 0$
$x = 0, \pi$	$\cos x = 1$	$\sin x = -1$
	$x = 0, \pi$	$x = \frac{3\pi}{2}$

$x = 0, \pi, \frac{3\pi}{2}$

possible solutions