Practice Integrals - Mixed

① $\int x^2 e^{2x}\,dx = \boxed{\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C}$

u		dv
x^2	$+$	e^{2x}
$2x$	$-$	$\frac{1}{2}e^{2x}$
2	$+$	$\frac{1}{4}e^{2x}$
0		$\frac{1}{8}e^{2x}$

② $\int_0^5 x\sqrt[3]{x^2+2}\,dx = \int \frac{1}{2}u^{1/3}\,du$

$u = x^2+2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\left(\frac{3}{4}\right)u^{4/3}$$

$$= \frac{3}{8}(x^2+2)^{4/3}\Big|_0^5$$

$$= \frac{3}{8}(5^2+2)^{4/3} - \frac{3}{8}(0^2+2)^{4/3}$$

$$= \frac{3}{8}(81) - \frac{3}{8}(2)^{4/3}$$

$$\boxed{\approx 29.43}$$

③ $\int (8x+7)\ln(5x)\,dx = (4x^2+7x)\ln(5x) - \int (4x^2+7x)(5)\left(\frac{1}{x}\right)dx$

$u = \ln(5x)$ $\quad v = 4x^2+7x$

$du = 5\left(\frac{1}{x}\right)dx$ $\quad dv = (8x+7)\,dx$

$$= (4x^2+7x)\ln(5x) - \int (20x+35)\,dx$$

$$\boxed{= (4x^2+7x)\ln(5x) - 10x^2 - 35x + C}$$

④ $\displaystyle\int_0^1 \frac{x^2}{2x^3+1}\,dx = \int \frac{1}{6}\frac{du}{u}$

$\displaystyle = \frac{1}{6}\ln|u| = \frac{1}{6}\ln|2x^3+1|\Big|_0^1$

$u = 2x^3+1$

$du = 6x^2dx$

$\frac{1}{6}du = x^2dx$

$\displaystyle = \frac{1}{6}\ln|2(1)^3+1| - \frac{1}{6}\ln|2(0)^3+1|$

$\displaystyle = \frac{1}{6}\ln 3 - \frac{1}{6}\ln 1$

$\boxed{= \frac{1}{6}\ln 3 \approx 0.183}$

⑤ $\displaystyle\int x e^{x^2}dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$

$\boxed{= \frac{1}{2}e^{x^2} + C}$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

⑥ $\displaystyle\int_0^1 \frac{x^3}{\sqrt{3+x^2}}\,dx = \int_0^1 \frac{x^2\cdot x\,dx}{\sqrt{3+x^2}} = \int_0^1 \frac{(u-3)(\frac{1}{2})\,du}{u^{1/2}}$

$u = 3+x^2 \Rightarrow x^2 = u-3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\displaystyle = \frac{1}{2}\int_0^1 \left(u^{1-\frac{1}{2}} - 3u^{-1/2}\right) du$

$\displaystyle = \frac{1}{2}\int_0^1 \left(u^{1/2} - 3u^{-1/2}\right) du$

$\displaystyle = \frac{1}{2}\left(\frac{u^{3/2}}{\frac{3}{2}} - 3\frac{u^{1/2}}{\frac{1}{2}}\right)\Big|_0^1$

$\displaystyle = \frac{1}{3}(3+x^2)^{3/2} - 3(3+x^2)^{1/2}\Big|_0^1$

$\displaystyle = \frac{1}{3}(3+1^2)^{3/2} - 3(3+1^2)^{1/2} - \Big[\frac{1}{3}(3+0^2)^{3/2} - 3(3+0^2)^{1/2}$

$\displaystyle = \frac{8}{3} - 6 - \frac{1}{3}(3)^{3/2} + 3(3)^{1/2}$

$\boxed{= 0.1307}$

⑦ $$\int_1^2 (1-x^2)e^{2x}\,dx = \frac{1}{2}(1-x^2)e^{2x} + \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\Big|_1^2$$

$$= e^{2x}\left(\frac{1}{2} - \frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{4}\right)\Big|_1^2$$

$$= e^{2x}\left(-\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{4}\right)\Big|_1^2$$

$$= e^4\left(-\frac{1}{2}(4) + \frac{1}{2}(2) + \frac{1}{4}\right) - e^2\left(-\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right)$$

$$\boxed{= -\frac{3}{4}e^4 - \frac{1}{4}e^2}$$

$u = 1-x^2$

$du = -2x\,dx$ $\quad \cancel{dv} =$

u		dv
$1-x^2$		e^{2x}
	$+$	
$-2x$		$\frac{1}{2}e^{2x}$
	$-$	
-2		$\frac{1}{4}e^{2x}$
	$+$	
0		$\frac{1}{8}e^{2x}$

⑧ $$\int (2x-1)\ln(3x)\,dx = 3(x^2-x)\ln 3x - \int (x^2-x)\left(\frac{3}{x}\right)dx$$

$$= 3(x^2-x)\ln 3x - \int (3x-3)\,dx$$

$$\boxed{= (x^2-x)\ln 3x - \frac{3}{2}x^2 + 3x + C}$$

$u = \ln 3x$ $\qquad v = x^2 - x$

$du = 3\frac{1}{x}\,dx$ $\qquad dv = (2x-1)\,dx$

⑨ $$\int x^2\sqrt{x+2}\,dx = \frac{2}{3}x^2(x+2)^{3/2} - \frac{8}{15}x(x+2)^{5/2} + \frac{16}{105}(x+2)^{7/2} + C$$

can simplify further

$$= \frac{2}{105}(x+2)^{3/2}\left(35x^2 - 28x(x+2) + 8\overbrace{(x+2)^2}^{x^2+4x+4}\right) + C$$

$$= \frac{2}{105}(x+2)^{3/2}(35x^2 - 28x^2 - 56x + 8x^2 + 32x + 32) + C$$

$$\boxed{= \frac{2}{105}(x+2)^{3/2}(15x^2 - 24x + 32) + C}$$

u		dv
x^2		$(x+2)^{1/2}$
	$+$	
$2x$		$\frac{2}{3}(x+2)^{3/2}$
	$-$	
2		$\left(\frac{2}{3}\right)\left(\frac{2}{5}\right)(x+2)^{5/2}$
	$+$	
0		$\left(\frac{2}{3}\right)\left(\frac{2}{5}\right)\left(\frac{2}{7}\right)(x+2)^{7/2}$