Calculus Name_____ ID: 1

Review Assignment: Ch2 & Ch3 (partial)

Use algebra to evaluate each limit.

1) $\lim\limits_{x \to -\frac{\pi}{6}} \cos(2x)$

$\frac{1}{2}$

2) $\lim\limits_{x \to -\frac{\pi}{6}} -\csc(x)$

2

3) $\lim\limits_{x \to -\frac{3\pi}{4}} 2\cot(x)$

2

4) $\lim\limits_{x \to \frac{5\pi}{6}} -2\cot(2x)$

$\frac{2\sqrt{3}}{3}$

5) $\lim\limits_{x \to 3^+} \frac{|-x+3|}{-x+3}$

-1

6) $\lim\limits_{x \to -2^+} f(x),\ f(x) = \begin{cases} 2x+1, & x < -2 \\ x^2 + 2x, & x \geq -2 \end{cases}$

0

7) $\lim\limits_{x \to 1^-} f(x),\ f(x) = \begin{cases} 3, & x \leq 1 \\ x^2 - 6x + 8, & x > 1 \end{cases}$

3

8) $\lim\limits_{x \to 0^+} \frac{|x|}{x}$

1

9) $\lim_{x \to 2} -\frac{x-2}{x^2-5x+6}$

1

10) $\lim_{x \to -3} -\frac{x+3}{x^2+5x+6}$

1

11) $\lim_{x \to 2} -\frac{x-2}{x^2-x-2}$

$-\frac{1}{3}$

12) $\lim_{x \to -3} -\frac{x^2-9}{x+3}$

6

13) $\lim_{x \to 1^-} \frac{x+3}{x^2-2x+1}$

∞

14) $\lim_{x \to 1^+} \frac{x+1}{x^2-1}$

∞

15) $\lim_{x \to -\infty} -\frac{2x^2}{x^2-4}$

-2

16) $\lim_{x \to -\infty} \frac{2x^3}{2x^2+3}$

$-\infty$

Find the intervals on which each function is continuous.

17) $f(x) = \dfrac{x^2 - 5x + 6}{x - 3}$

$(-\infty, 3), (3, \infty)$

18) $f(x) = \dfrac{x}{x^2 - 2x + 1}$

$(-\infty, 1), (1, \infty)$

19) $f(x) = \dfrac{x - 4}{x^2 + 3x}$

$(-\infty, -3), (-3, 0), (0, \infty)$

20) $f(x) = \begin{cases} -2 - \dfrac{x}{2}, & x \neq 0 \\ -3, & x = 0 \end{cases}$

$(-\infty, 0), (0, \infty)$

Determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity.

21) $f(x) = \dfrac{x}{x^2 - 3x}$

Removable discontinuity at: $x = 0$
Essential discontinuity at: $x = 3$

22) $f(x) = \dfrac{x + 5}{x^2 - x - 2}$

Essential discontinuities at: $x = -1$, $x = 2$

23) $f(x) = \begin{cases} x - 3, & x < 1 \\ 2x - 6, & x \geq 1 \end{cases}$

Jump discontinuity at: $x = 1$

24) $f(x) = \begin{cases} -x^2 + 4x - 1, & x \neq 1 \\ 1, & x = 1 \end{cases}$

Removable discontinuity at: $x = 1$

Use the definition of the derivative to write the equation for dy/dx, but do not solve it.

25) $y = -5x - 2$

$\frac{dy}{dx} = -5$

26) $y = 2x^2 + 2x - 4$

$\frac{dy}{dx} = 4x + 2$

27) $y = -2x^2 - 1$

$\frac{dy}{dx} = -4x$

28) $y = -2x + 1$

$\frac{dy}{dx} = -2$

Differentiate each function with respect to *x*.

29) $y = x^3 - \frac{2}{3}x^{\frac{5}{2}} + 2x^{-3}$

$\frac{dy}{dx} = 3x^2 - \frac{5x^{\frac{3}{2}}}{3} - \frac{6}{x^4}$

30) $y = -\frac{1}{4}x + \frac{1}{4}x^{-2} + \frac{2}{3}x^{-3}$

$\frac{dy}{dx} = -\frac{1}{4} - \frac{1}{2x^3} - \frac{2}{x^4}$

31) $y = 3x^4(-2x^4 - 1)$

$\frac{dy}{dx} = 3x^4 \cdot -8x^3 + (-2x^4 - 1) \cdot 12x^3$

$= -48x^7 - 12x^3$

32) $y = (x^5 + 5)x^4$

$\frac{dy}{dx} = (x^5 + 5) \cdot 4x^3 + x^4 \cdot 5x^4$

$= 9x^8 + 20x^3$

33) $y = \dfrac{5x^5}{3x^4 + 2}$

$$\frac{dy}{dx} = \frac{(3x^4 + 2) \cdot 25x^4 - 5x^5 \cdot 12x^3}{(3x^4 + 2)^2}$$
$$= \frac{15x^8 + 50x^4}{9x^8 + 12x^4 + 4}$$

34) $y = \dfrac{x^4}{x^2 - 2}$

$$\frac{dy}{dx} = \frac{(x^2 - 2) \cdot 4x^3 - x^4 \cdot 2x}{(x^2 - 2)^2}$$
$$= \frac{2x^5 - 8x^3}{x^4 - 4x^2 + 4}$$

35) $y = \dfrac{4}{2x^2 - 5}$

$$\frac{dy}{dx} = -\frac{4 \cdot 4x}{(2x^2 - 5)^2}$$
$$= -\frac{16x}{4x^4 - 20x^2 + 25}$$

36) $y = \dfrac{2x^3}{2x^4 + 5}$

$$\frac{dy}{dx} = \frac{(2x^4 + 5) \cdot 6x^2 - 2x^3 \cdot 8x^3}{(2x^4 + 5)^2}$$
$$= \frac{-4x^6 + 30x^2}{4x^8 + 20x^4 + 25}$$

37) $y = (5x + 4)^5$

$$\frac{dy}{dx} = 5(5x + 4)^4 \cdot 5$$
$$= 25(5x + 4)^4$$

38) $y = (4x^5 - 1)^{-2}$

$$\frac{dy}{dx} = -2(4x^5 - 1)^{-3} \cdot 20x^4$$
$$= -\frac{40x^4}{(4x^5 - 1)^3}$$

39) $y = (-2x^4 + 5)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(-2x^4 + 5)^{-\frac{1}{2}} \cdot -8x^3$$
$$= -\frac{4x^3}{(-2x^4 + 5)^{\frac{1}{2}}}$$

40) $y = (-2x^2 - 1)^{-2}$

$$\frac{dy}{dx} = -2(-2x^2 - 1)^{-3} \cdot -4x$$
$$= \frac{8x}{(-2x^2 - 1)^3}$$

41) $y = \sin(x^3 - 4)^3$

$$\frac{dy}{dx} = \cos(x^3 - 4)^3 \cdot 3(x^3 - 4)^2 \cdot 3x^2$$
$$= 9x^2 \cos(x^3 - 4)^3 \cdot (x^3 - 4)^2$$

42) $y = \csc \sqrt[3]{x^5 - 2}$

$$\frac{dy}{dx} = -\csc\left(x^5 - 2\right)^{\frac{1}{3}} \cot\left(x^5 - 2\right)^{\frac{1}{3}} \cdot \frac{1}{3}\left(x^5 - 2\right)^{-\frac{2}{3}} \cdot 5x^4$$

$$= -\frac{5x^4 \csc\left(x^5 - 2\right)^{\frac{1}{3}} \cot\left(x^5 - 2\right)^{\frac{1}{3}}}{3\left(x^5 - 2\right)^{\frac{2}{3}}}$$

43) $y = \sqrt[3]{\cos 2x^5}$

$$\frac{dy}{dx} = \frac{1}{3} \cdot \left(\cos 2x^5\right)^{-\frac{2}{3}} \cdot -\sin 2x^5 \cdot 10x^4$$

$$= -\frac{10x^4 \sin 2x^5}{3\left(\cos 2x^5\right)^{\frac{2}{3}}}$$

44) $y = \cos \sqrt{-5x^3 - 1}$

$$\frac{dy}{dx} = -\sin\left(-5x^3 - 1\right)^{\frac{1}{2}} \cdot \frac{1}{2}\left(-5x^3 - 1\right)^{-\frac{1}{2}} \cdot -15x^2$$

$$= \frac{15x^2 \sin\left(-5x^3 - 1\right)^{\frac{1}{2}}}{2\left(-5x^3 - 1\right)^{\frac{1}{2}}}$$

45) $y = \ln 5x^5$

$$\frac{dy}{dx} = \frac{1}{5x^5} \cdot 25x^4$$

$$= \frac{5}{x}$$

46) $y = e^{5x^5}$

$$\frac{dy}{dx} = e^{5x^5} \cdot 25x^4$$

47) $y = \ln 5x^3$

$$\frac{dy}{dx} = \frac{1}{5x^3} \cdot 15x^2$$

$$= \frac{3}{x}$$

48) $y = e^{x^2}$

$$\frac{dy}{dx} = e^{x^2} \cdot 2x$$

49) $y = \sin^{-1} -4x^2$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(-4x^2\right)^2}} \cdot -8x$$

$$= -\frac{8x}{\sqrt{1 - 16x^4}}$$

50) $y = \sin^{-1} 4x^3$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(4x^3\right)^2}} \cdot 12x^2$$

$$= \frac{12x^2}{\sqrt{1 - 16x^6}}$$